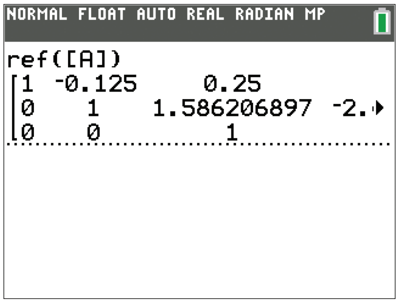
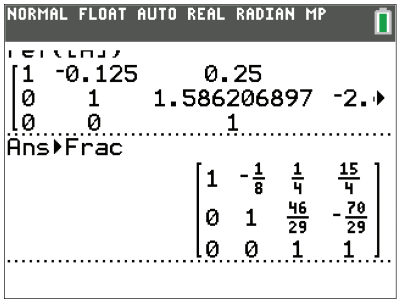
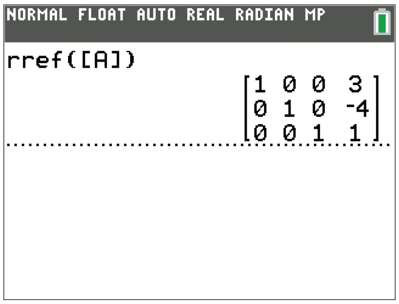
Return to the main screen by pressing **2nd** **mode**. Now that the matrix is stored in the calculator as matrix *A*, we can perform operations. Press **2nd** **x⁻¹** to access the MATRIX menu again, but this time select MATH. Scroll down to select A:ref(. When you press **enter**, ref(will appear on the main screen.

Now we need to select which matrix we want to put in row echelon form, matrix *A*. To do this, press **2nd** **x⁻¹** once more. NAMES and 1:[A] should already be highlighted, so press **enter**. Add the right-hand parenthesis and press **enter**.

To view this matrix with fractional entries, press **math** and **enter**, since 1:Frac is already highlighted. Press **enter** again on the main screen.







Since the entire matrix does not fit on the screen, use the arrow keys to view the right side of the matrix.

Finding the reduced row echelon form of this matrix is similar. Press **2nd** **x⁻¹** and select MATH but this time arrow down farther to highlight B:rref(and press **enter**. Select matrix *A* again, add the right-hand parenthesis and press **enter**.

6.2 EXERCISES

PRACTICE

1. Let $A = \begin{bmatrix} 4 & -1 \\ 0 & 3 \\ 9 & -5 \end{bmatrix}$. Determine the following, if possible:

a. The order of *A* **b.** The value of a_{12} **c.** The value of a_{23}

2. Let $B = \begin{bmatrix} -7 & 2 & 11 \end{bmatrix}$. Determine the following, if possible:

a. The order of *B* **b.** The value of b_{12} **c.** The value of b_{31}

3. Let $C = \begin{bmatrix} 1 & 0 \\ 5 & -3 \\ 2 & 9 \\ \pi & e \\ 10 & -7 \end{bmatrix}$. Determine the following, if possible:

a. The order of C **b.** The value of c_{23} **c.** The value of c_{51}

4. Let $D = \begin{bmatrix} -8 & 13 & -1 \\ 0 & 6 & 3 \\ 0 & -9 & 0 \end{bmatrix}$. Determine the following, if possible:

a. The order of D **b.** The value of d_{23} **c.** The value of d_{33}

5. Let $E = \begin{bmatrix} -443 & 951 & 165 & 274 \\ 286 & -653 & 812 & -330 \\ 909 & 377 & 429 & -298 \end{bmatrix}$. Determine the following, if possible:

a. The order of E **b.** The value of e_{42} **c.** The value of e_{21}

6. Let $A = \begin{bmatrix} 9 & 5 & 0 \\ 7 & 4 & 2 \end{bmatrix}$. Determine the following, if possible:

a. The order of A **b.** The value of a_{22} **c.** The value of a_{13}

7. Let $B = \begin{bmatrix} 8 & 1 \\ 3 & 0 \\ 6 & 7 \end{bmatrix}$. Determine the following, if possible:

a. The order of B **b.** The value of b_{12} **c.** The value of b_{13}

8. Let $C = \begin{bmatrix} 65 & 32 & 91 & 45 \\ 23 & 18 & 75 & 47 \\ 8 & 63 & 28 & 31 \end{bmatrix}$. Determine the following, if possible:

a. The order of C **b.** The value of c_{43} **c.** The value of c_{23}

9. Let $D = \begin{bmatrix} 4 & 9 & 7 & 1 & 8 \\ 5 & 3 & 0 & 2 & 6 \end{bmatrix}$. Determine the following, if possible:

a. The order of D **b.** The value of d_{21} **c.** The value of d_{24}

Construct the augmented matrix that corresponds to each of the following systems of equations. See Example 2. (Answers may appear in slightly different, but equivalent, form.)

10. $\begin{cases} 4x + 5y - 3z = 8 \\ 7x - 2y + 9 = 3 \\ 5x - 6y + 3z = 0 \end{cases}$

11. $\begin{cases} y - 2z + 4 = 3x \\ \dfrac{x}{2} - 4y - 1 = z \\ 3(-y + z) - 1 = 0 \end{cases}$

12. $\begin{cases} 5x+\dfrac{y-z}{2}=3 \\ 7(z-x)+y-2=0 \\ x-(4-z)=y \end{cases}$

13. $\begin{cases} \dfrac{2-3x}{2}=y \\ 3z+2(x+y)=0 \\ 2x-y=2(x-3z) \end{cases}$

14. $\begin{cases} 2(z+3)-x+y=z \\ -3(x-2y)-1=5z \\ \dfrac{x}{3}-(y-2z)=x \end{cases}$

15. $\begin{cases} \dfrac{12x-1}{5}+\dfrac{y}{2}=\dfrac{3z}{2} \\ y-(x+3z)=-(1-y) \\ 2x-2-z-2y=7x \end{cases}$

16. $\begin{cases} \dfrac{3x+4y}{2}-3z=6 \\ 3(x-2y+9z)=0 \\ 2x+6y=3-z \end{cases}$

17. $\begin{cases} \dfrac{2x-4y}{3}=2z \\ 8x=2(y-3z)+7 \\ 3x=2y \end{cases}$

18. $\begin{cases} \dfrac{2(2x-y)}{3}+z=7 \\ 4=\dfrac{3}{-x+y+3z} \\ 4x-8y+4=9x \end{cases}$

19. $\begin{cases} 0.5x-14y=\dfrac{z}{4}-8 \\ \dfrac{x}{5}-y+\dfrac{z}{4}=\dfrac{y}{6}-3 \\ \dfrac{2}{3}\left(\dfrac{4}{y-x-1}\right)=\dfrac{5}{z} \end{cases}$

Construct the system of equations that corresponds to each of the following matrices.

20. $\left[\begin{array}{cc|c} 5 & 3 & 9 \\ 1 & 4 & 12 \end{array}\right]$

21. $\left[\begin{array}{cc|c} 1 & 0 & 8 \\ 0 & 1 & 3 \end{array}\right]$

22. $\left[\begin{array}{ccc|c} 14 & 0 & 1 & 16 \\ 3 & 6 & 4 & 0 \\ 8 & 2 & 5 & 21 \end{array}\right]$

23. $\left[\begin{array}{ccc|c} 1 & 3 & 6 & 16 \\ 0 & 1 & 2 & 9 \\ 0 & 0 & 1 & 4 \end{array}\right]$

24. $\left[\begin{array}{ccc|c} 2 & 1 & 1 & 22 \\ 1 & 3 & 1 & 17 \\ 1 & 1 & 4 & 8 \end{array}\right]$

25. $\left[\begin{array}{ccc|c} 0 & 9 & 13 & 27 \\ 2 & 0 & 21 & 19 \\ 7 & 18 & 0 & 32 \end{array}\right]$

Fill in the blanks by performing the indicated row operations. See Example 4.

26. $\left[\begin{array}{cc|c} 3 & 2 & -7 \\ 1 & 3 & 5 \end{array}\right] \xrightarrow{-3R_2+R_1} \underline{?}$

27. $\left[\begin{array}{cc|c} 2 & -5 & 3 \\ -4 & 3 & -1 \end{array}\right] \xrightarrow{2R_1+R_2} \underline{?}$

28. $\left[\begin{array}{cc|c} 4 & 2 & -8 \\ 3 & -9 & 0 \end{array}\right] \xrightarrow[\frac{1}{3}R_2]{\frac{1}{2}R_1} \underline{?}$

29. $\left[\begin{array}{cc|c} 9 & -2 & 7 \\ 1 & 3 & -2 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \underline{?}$

30. $\left[\begin{array}{cc|c} 4 & 1 & 5 \\ 3 & 6 & 0 \end{array}\right] \xrightarrow{2R_1} \underline{?}$

31. $\left[\begin{array}{cc|c} 8 & -2 & -4 \\ 3 & -1 & 7 \end{array}\right] \xrightarrow{-2R_2} \underline{?}$

32. $\left[\begin{array}{cc|c} 9 & 12 & -6 \\ 15 & -3 & 0 \end{array}\right] \xrightarrow{-\frac{1}{3}R_1} \underline{?}$

33. $\left[\begin{array}{cc|c} 4 & 12 & -6 \\ 7 & 3 & 9 \end{array}\right] \xrightarrow{\frac{1}{2}R_1+R_2} \underline{?}$

34. $\left[\begin{array}{cc|c} 3 & 0 & 1 \\ 5 & 7 & -2 \end{array}\right] \xrightarrow{3R_1+R_2} \underline{?}$

35. $\left[\begin{array}{cc|c} 8 & -2 & 10 \\ 9 & -3 & 0 \end{array}\right] \xrightarrow[-\frac{2}{3}R_2]{\frac{1}{2}R_1} \underline{?}$

36. $\left[\begin{array}{ccc|c} 5 & 2 & 9 & 7 \\ 1 & 3 & -5 & 0 \\ 2 & -4 & 1 & 8 \end{array}\right] \xrightarrow[-R_1+R_3]{2R_2} \underline{\ ?\ }$

37. $\left[\begin{array}{ccc|c} 6 & -2 & 5 & 14 \\ -7 & 19 & 2 & 3 \\ -9 & 11 & -4 & 7 \end{array}\right] \xrightarrow[0.5R_3]{3R_1} \underline{\ ?\ }$

38. $\left[\begin{array}{ccc|c} 5 & 3 & 13 & 15 \\ 17 & 9 & -8 & -14 \\ 4 & -11 & 19 & 8 \end{array}\right] \xrightarrow{-2R_2+R_3} \underline{\ ?\ }$

39. $\left[\begin{array}{ccc|c} 8 & 11 & 18 & 2 \\ 14 & 33 & -3 & -5 \\ -9 & 21 & 12 & 9 \end{array}\right] \xrightarrow[-2R_3+R_2]{\frac{1}{3}R_3+R_1} \underline{\ ?\ }$

40. $\left[\begin{array}{ccc|c} 1 & 3 & -2 & 4 \\ 3 & -1 & 8 & 2 \\ -5 & 0 & 2 & 7 \end{array}\right] \xrightarrow[5R_1+R_3]{-3R_1+R_2} \underline{\ ?\ }$

41. $\left[\begin{array}{ccc|c} 2 & 3 & -3 & 5 \\ 1 & 1 & 3 & 4 \\ 3 & 3 & 9 & 12 \end{array}\right] \xrightarrow[-3R_2+R_3]{-2R_2+R_1} \underline{\ ?\ }$

42. $\left[\begin{array}{cc|c} -3 & 2 & 2 \\ 5 & -4 & 1 \end{array}\right] \xrightarrow{2R_1+R_2} \underline{\ ?\ }$

43. $\left[\begin{array}{cc|c} -5 & 20 & -15 \\ 2 & -12 & 5 \end{array}\right] \xrightarrow[\frac{1}{2}R_2]{\frac{1}{5}R_1} \underline{\ ?\ }$

44. $\left[\begin{array}{ccc|c} 2 & 2 & 3 & 7 \\ -3 & 2 & 8 & -2 \\ 1 & 5 & 2 & 6 \end{array}\right] \xrightarrow[3R_3+R_2]{-2R_3+R_1} \underline{\ ?\ }$

45. $\left[\begin{array}{ccc|c} 1 & 5 & -9 & 11 \\ 1 & 4 & -1 & 4 \\ 4 & 3 & 5 & 45 \end{array}\right] \xrightarrow[-4R_1+R_3]{-R_1+R_2} \underline{\ ?\ }$

For each matrix, determine if it is in row echelon form, reduced row echelon form, or neither.

46. $\left[\begin{array}{cc|c} 1 & 5 & 4 \\ 0 & 1 & 3 \end{array}\right]$

47. $\left[\begin{array}{ccc|c} 1 & 2 & 0 & 9 \\ 0 & 1 & 3 & 4 \\ 0 & 1 & 1 & 12 \end{array}\right]$

48. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 8 \end{array}\right]$

49. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 5 & 1 & 0 & 14 \\ 3 & 4 & 1 & -16 \end{array}\right]$

50. $\left[\begin{array}{ccc|c} 1 & 2 & 5 & 0 \\ 0 & 1 & 9 & 3 \\ 0 & 0 & 0 & 1 \end{array}\right]$

51. $\left[\begin{array}{cc|c} 0 & 1 & 3 \\ 1 & 0 & 6 \end{array}\right]$

Use Gaussian elimination and back-substitution to solve the following systems of equations. See Example 5.

52. $\begin{cases} 2x-4y=-6 \\ 3x-y=-4 \end{cases}$

53. $\begin{cases} 2x-5y=11 \\ 3x+2y=7 \end{cases}$

54. $\begin{cases} 5x-y=-21 \\ 9x+2y=-34 \end{cases}$

55. $\begin{cases} x-4y=-11 \\ 7x-y=4 \end{cases}$

56. $\begin{cases} x+2y=17 \\ 3x+4y=39 \end{cases}$

57. $\begin{cases} 2x+6y=4 \\ -4x-7y=7 \end{cases}$

58. $\begin{cases} 3x-2y=5 \\ -5x+4y=-3 \end{cases}$

59. $\begin{cases} 2x+y=-2 \\ -4x-2y=5 \end{cases}$

60. $\begin{cases} 6x-16y=10 \\ -3x+8y=4 \end{cases}$

61. $\begin{cases} 2x-3y=0 \\ 5x+y=17 \end{cases}$

62. $\begin{cases} 6x+3y=3 \\ x+y=3 \end{cases}$

63. $\begin{cases} 3x+6y=-12 \\ 2x+4y=-8 \end{cases}$

64. $\begin{cases} 4x+5y=9 \\ 8x+3y=-17 \end{cases}$

65. $\begin{cases} \frac{2}{3}x+2y=1 \\ x+3y=0 \end{cases}$

66. $\begin{cases} 13x-17y=-3 \\ -19x+15y=-35 \end{cases}$

67. $\begin{cases} 3x-9y-7z=-9 \\ 5x+11y-z=17 \\ -4x-8y+7z=5 \end{cases}$ **68.** $\begin{cases} 8x-y+5z=-8 \\ 11x-2y+9z=-9 \\ 7x-3y+13z=4 \end{cases}$ **69.** $\begin{cases} 17x+13y+8z=46 \\ -12x+3y+28z=-19 \\ 14x+5y-15z=-15 \end{cases}$

Use Gauss-Jordan elimination to solve the following systems of equations. See Example 6.

70. $\begin{cases} 2x-3y=8 \\ 8x+5y=-2 \end{cases}$ **71.** $\begin{cases} \frac{2}{3}x+y=-3 \\ 3x+\frac{5}{2}y=-\frac{7}{2} \end{cases}$ **72.** $\begin{cases} 3y=9 \\ x+2y=11 \end{cases}$

73. $\begin{cases} 6x+2y=-4 \\ -9x-3y=6 \end{cases}$ **74.** $\begin{cases} 3y=6 \\ 5x+2y=4 \end{cases}$ **75.** $\begin{cases} 3x+8y=-4 \\ x+2y=-2 \end{cases}$

76. $\begin{cases} -3x+2y=5 \\ 5x-2y=1 \end{cases}$ **77.** $\begin{cases} 9x-11y=10 \\ -4x+3y=-12 \end{cases}$ **78.** $\begin{cases} 9x-15y=-6 \\ -3x+11y=-10 \end{cases}$

79. $\begin{cases} 3x-8y=7 \\ 18x-35y=-23 \end{cases}$ **80.** $\begin{cases} 4x+y-3z=-9 \\ 2x-3z=-19 \\ 7x-y-4z=-29 \end{cases}$ **81.** $\begin{cases} -5x+9y+3z=1 \\ 3x+2y-6z=9 \\ x+4y-z=16 \end{cases}$

82. $\begin{cases} 2x-y=0 \\ 5x-3y-3z=5 \\ 2x+6z=-10 \end{cases}$ **83.** $\begin{cases} x+y=4 \\ y+3z=-1 \\ 2x-2y+5z=-5 \end{cases}$ **84.** $\begin{cases} 2x-3y=-2 \\ x-4y+3z=0 \\ -2x+7y-5z=0 \end{cases}$

85. $\begin{cases} 3x+8z=3 \\ -3x-7z=-3 \\ x+3z=1 \end{cases}$ **86.** $\begin{cases} 3x-y+z=2 \\ -6x+2y-2z=1 \\ 5x+2y-3z=2 \end{cases}$ **87.** $\begin{cases} x+2y=-1 \\ y+3z=7 \\ 2x+5z=21 \end{cases}$

88. $\begin{cases} 2x+8y-z=-5 \\ -5x+3y+4z=-6 \\ x-4y-5z=-8 \end{cases}$ **89.** $\begin{cases} 7x-8y+2z=-2 \\ 5x-3y-z=-3 \\ 8x+y-3z=7 \end{cases}$

90. $\begin{cases} 8x+14y-3z=3 \\ -6x+2y+7z=-13 \\ 8x+19y+3z=11 \end{cases}$ **91.** $\begin{cases} 8x+5y+3z=-2 \\ 12x-y-18z=1 \\ 7x+6y+10z=19 \end{cases}$

92. $\begin{cases} 4x+8y+7z=27 \\ -2x+9y-8z=-15 \\ 9x+13y+7z=-33 \end{cases}$ **93.** $\begin{cases} w-x+2z=9 \\ 2w+3y=-1 \\ -2w-5y-z=0 \\ x+2y=-4 \end{cases}$

94. $\begin{cases} 3w-x+5y+3z=2 \\ -4w-10y-2z=10 \\ w-x+2z=7 \\ 4w-2x+5y+5z=9 \end{cases}$

APPLICATIONS

95. The sum of three integers is 155. The first integer is sixteen more than the second. The third integer is seven less than the sum of the first integer and twice the second. What are the three integers?

96. Mario bought a pound of bacon, a dozen eggs, and a loaf of bread to make breakfast for his family. The total cost was \$7.42. The bacon cost \$0.03 more than twice the price of the bread and the eggs cost \$0.03 less than half the price of the bread. Find the price of each item.

97. The Pizza House sells three sizes of pizzas: small, medium, large. The prices of the pizzas are \$9.00, \$12.00, and \$15.00, respectively. In one day, they sold 82 pizzas for a total of \$1098.00. If the number of large pizzas sold was twice the number of medium pizzas sold, how many of each size pizza did the Pizza House sell?