The Decibel Scale

In the **decibel scale**, I_0 is the intensity of a just-discernible sound, I is the intensity of the sound being analyzed, and D is its decibel level.

$$D = 10\log\left(\frac{I}{I_0}\right)$$

Decibel levels range from 0 for a barely discernible sound, to 60 for the level of normal conversation, to 80 for heavy traffic, to 120 for a loud rock concert, and finally (as far as humans are concerned) to around 160, at which point the eardrum is likely to rupture.

Example 8: The Decibel Scale

Given that $I_0 = 10^{-12}$ watts/meter^2, what is the decibel level of a jet airliner's engines at a distance of 45 meters, for which the sound intensity is 50 watts/meter^2?

Solution

$$\begin{aligned} D &= 10\log\left(\frac{50}{10^{-12}}\right) \\ &= 10\log\left(5\times10^{13}\right) \\ &= 10\left(\log 5 + 13\right) \\ &\approx 137 \end{aligned}$$

In other words, the sound level would probably not be literally earsplitting, but it would be very painful.

4.4 EXERCISES

PRACTICE

Use the properties of logarithms to expand the following expressions as much as possible. Simplify any numerical expressions that can be evaluated without a calculator. See Example 3.

1. $\log_5\left(125x^3\right)$

2. $\ln\left(\frac{x^2y}{3}\right)$

3. $\ln\left(\frac{e^2p}{q^3}\right)$

4. $\log\left(100x\right)$

5. $\log_9\left(9xy^{-3}\right)$

6. $\log_6\left(\sqrt[3]{\frac{p^2}{q}}\right)$

7. $\ln\left(\frac{\sqrt{x^3}pq^5}{e^7}\right)$

8. $\log_a\sqrt[5]{\frac{a^4b}{c^2}}$

9. $\log\left(\log\left(100x^3\right)\right)$

10. $\log_3\left(9x+27y\right)$

11. $\log\left(\frac{10}{\sqrt{x+y}}\right)$

12. $\ln\left(\ln\left(e^{ex}\right)\right)$

13. $\log_2\left(\dfrac{y^2+z}{16x^4}\right)$ **14.** $\log\left(\log\left(100{,}000^{2x}\right)\right)$ **15.** $\log_b\left(\sqrt{\dfrac{x^4y}{z^2}}\right)$

16. $\ln\left(7x^2-42x+63\right)$ **17.** $\log_b\left(ab^2c^b\right)$ **18.** $\ln\left(\ln\left(e^{e^x}\right)\right)$

Use the properties of logarithms to condense the following expressions as much as possible, writing each answer as a single term with a coefficient of 1. See Example 4.

19. $\log x-\log y$ **20.** $\log_5 x-2\log_5 y$

21. $\log_5\left(x^2-25\right)-\log_5\left(x-5\right)$ **22.** $\ln\left(x^2y\right)-\ln y-\ln x$

23. $\dfrac{1}{3}\log_2 x+\log_2\left(x+3\right)$ **24.** $\dfrac{1}{5}\left(\log_7\left(x^2\right)-\log_7\left(pq\right)\right)$

25. $\ln 3+\ln p-2\ln q$ **26.** $2\left(\log_5\left(\sqrt{x}\right)-\log_5 y\right)$

27. $\log\left(x-10\right)-\log x$ **28.** $2\log a^2b-\log\left(\dfrac{1}{b}\right)+\log\left(\dfrac{1}{a}\right)$

29. $3\left(\ln\left(\sqrt[3]{z^2}\right)-\ln\left(xy\right)\right)$ **30.** $\log_2\left(4x\right)-\log_2 x$

31. $\log_5 20-\log_5 5$ **32.** $\log 30-\log 2-\log 5$

33. $\ln 15+\ln 3$ **34.** $\ln 8-\ln 4+\ln 3$

35. $0.5\log_3 16-\log_3 4$ **36.** $3\log_7 2-2\log_7 4$

37. $0.25\ln 81+\ln 4$ **38.** $2\left(\log 4-\log 1+\log 2\right)$

39. $\log 11+0.5\log 9-\log 3$ **40.** $3\log_4\left(x^2\right)+\log_4\left(x^6\right)$

41. $\log_8\left(2x^2-2y\right)-0.25\log_8 16$ **42.** $\log_{3x} x^2+\log_{3x} 18-\log_{3x} 6$

Use the properties of logarithms to write each of the following as a single term that does not contain a logarithm.

43. $5^{2\log_5 x}$ **44.** $10^{\log y^2-3\log x}$ **45.** $e^{2-\ln x+\ln p}$

46. $e^{5\left(\ln\sqrt[5]{3}+\ln x\right)}$ **47.** $10^{\log x^3-4\log y}$ **48.** $a^{\log_a b+4\log_a\sqrt{a}}$

49. $10^{2\log x}$ **50.** $10^{4\log x-2\log x}$ **51.** $\log_4 16\cdot\log_x x^2$

52. $e^{\ln x+2+\ln x^2}$ **53.** $4^{\log_4(3x)+0.5\log_4\left(16x^2\right)}$ **54.** $4^{2\log_2 6-\log_2 9}$

Evaluate the following logarithmic expressions. See Example 5.

55. $\log_4 17$ **56.** $2\log_{\frac{1}{3}} 5$ **57.** $\log_9 8$

58. $\log_2 0.01$ **59.** $\log_{12} 10.5$ **60.** $\log\left(\ln 2\right)$

61. $\log_6 3^4$ **62.** $\log_7 14.3$ **63.** $\log_{\frac{1}{2}}\pi^{-2}$

64. $\log_{\frac{1}{5}} 626$ **65.** $\ln\left(\log 123\right)$ **66.** $\log_{17} 0.041$

67. $\log 16$

68. $\log_3 9$

69. $\log_5 20$

70. $\log_8 26$

71. $\log_4 0.25$

72. $\log_{1.8} 9$

73. $\log_{2.5} 34$

74. $\log_{0.5} 10$

75. $\log_4 2.9$

76. $\log_{0.4} 14$

77. $\log_{0.2} 17$

78. $\log_{0.16} 2.8$

Without using a calculator, evaluate the following expressions.

79. $\log_4 16$

80. $\log_5 25^3$

81. $\ln e^4 + \ln e^3$

82. $\log_4 \frac{1}{64}$

83. $\ln e^{1.5} - \log_4 2$

84. $\log_2 8^{(2\log_2 4 - \log_2 4)}$

Find the value of *x* in each of the following equations. Express your answer in exact form, or rounded to two decimal places.

85. $\log_x 1024 = 4$

86. $\log_6 729 = x$

87. $\log_2 529 = x$

88. $\log_4 625 = x$

89. $\log_x 729 = 9$

90. $\log_4 x = 8$

91. $\log_{12} x = 1$

92. $\log_x 16,807 = 7$

93. $\log_4 x = 10$

APPLICATIONS

94. A certain brand of tomato juice has a $\left[H_3O^+\right]$ concentration of 6.31×10^{-5} moles/liter. What is the pH of this brand?

95. One type of detergent, when added to neutral water with a pH of 7, results in a solution with a $\left[H_3O^+\right]$ concentration that is 5.62×10^{-4} times weaker than that of the water. What is the pH of the solution?

96. What is the concentration of $\left[H_3O^+\right]$ in lemon juice with a pH of 2.1?

97. An earthquake in Chile in 2019 measured 6.7 on the Richter scale. What was the intensity, relative to a 0-level earthquake, of this event?

98. How much stronger was the 2001 Gujarat earthquake (6.9 on the Richter scale) than the 2019 earthquake described in Exercise 97?

99. A construction worker operating a jackhammer would experience noise with an intensity of 20 watts/meter^2 if it weren't for ear protection. Given that $I_0 = 10^{-12}$ watts/meter^2, what is the decibel level for such noise?

100. A microphone picks up the sound of a thunderclap and measures its decibel level as 105. Given that $I_0 = 10^{-12}$ watts/meter^2, with what sound intensity did the thunderclap reach the microphone?

101. Matt, a lifeguard, has to make sure that the pH of the swimming pool stays between 7.2 and 7.6. If the pH is out of this range, he has to add chemicals that alter the pH level of the pool. If Matt measures the $\left[H_3O^+\right]$ concentration in the swimming pool to be 2.40×10^{-8} moles/liter, what is the pH? Does he need to change the pH by adding chemicals to the water?

102. The intensity of a cat's soft purring is measured to be 2.19×10^{-11}. Given that $I_0=10^{-12}$ watts/meter^2, what is the decibel level of this noise?

103. Newton's Law of Cooling states that the rate at which an object cools is proportional to the difference between the temperature of the object and the surrounding temperature. If C denotes the surrounding temperature and T_0 denotes the temperature at time $t = 0$, the temperature of an object at time t is given by $T(t)=C+(T_0-C)e^{-kt}$, where k is a constant that depends on the particular object under discussion.

a. You are having friends over for tea and want to know how long after boiling the water it will be drinkable. If the temperature of your kitchen stays around 74°F and you found online that the constant k for tea is approximately 0.049, how many minutes after boiling the water will the tea be drinkable (you prefer your tea no warmer than 140°F)? Recall that water boils at 212°F.

b. As you intern for your local crime scene investigation department, you are asked to determine at what time a victim died. If you are told k is approximately 0.1947 for a human body and the body's temperature was 72°F at 1:00 a.m., and the body has been in a storage building at a constant 60°F, approximately what time did the victim die? Recall the average temperature for a human body is 98.6°F. Note in this situation, t is measured in hours.

c. When helping your father cook a turkey, you were told to remove the turkey when the thickest part had reached 180°F. If you remove the turkey and place it on the table in a room that is 72°F, and it cools to 155°F in 20 minutes, what will the temperature of the turkey be at lunch time (an hour and 15 minutes after the turkey is removed from the oven)? Should you warm the turkey before eating?