Solution

Again, the solution boils down to substituting the correct values and evaluating the result. Here, $P = 10{,}000$, $r = 0.045$, and $t = 3.5$.

$$\begin{aligned} A(3.5) &= 10{,}000e^{(0.045)(3.5)} \\ &= 10{,}000e^{0.1575} \\ &\approx \$11{,}705.81 \end{aligned}$$

This account earns \$10.29 more than the quarterly compounded account in Example 3.

All exponential functions can be expressed with the base e (or any other base, for that matter). The base e is so commonly used for exponential functions that it is often called the *natural base*. For instance, the formula for the radioactive decay of carbon-14, using the base e, is

$$A(t) = A_0e^{-0.000121}t.$$

You should verify that this version of the decay formula does indeed give the same values for $A(t)$ as the version derived in Example 2.

4.2 EXERCISES

APPLICATIONS

1. A new virus has broken out in isolated parts of Africa and is spreading exponentially through tribal villages. The growth of this new virus can be mapped using the following formula where V stands for the number of people in the village who are infected with the virus, P stands for the number of people in a village and d stands for the number of days since the virus first appeared. According to this equation, how many people in a village of 300 will be infected after 5 days?

$$V = P\left(1-e^{-0.18d}\right)$$

2. A prototype for an electric motorcycle uses a battery whose energy capacity $C(d)$, in kilowatt-hours (kWh), is given by the formula $C(d) = 12e^{-0.02d}$, where d represents the number of days since receiving a full charge. What is the battery's energy capacity 30 days after being fully charged?

3. A young economics student has come across a very profitable investment scheme in which his money will accrue interest according to the equation listed below, where C represents the investment value after m months for an initial investment of I dollars. If this student invests \$1250 into this lucrative endeavor, how much money will he have after 24 months? I represents the investment and m represents the number of months the money has been invested for.

$$C = Ie^{0.08m}$$

4. A family releases a couple of pet rabbits into the wild. Upon being released the rabbits begin to reproduce at an exponential rate, as shown in the formula below. After 2 years how large is the rabbit population P, where n stands for the initial rabbit population (2) and m stands for the number of months?

$$P = ne^{0.5m}$$

5. Inside a business network, an email worm was downloaded by an employee. This worm goes through the infected computer's address book and sends itself to all the listed email addresses. This worm very rapidly works its way through the network following the equation below, where C is the number of computers in the network and W is the number of computers infected h hours after the worm is initially downloaded. After only 8 hours, how many computers has the worm infected if there are 150 computers in the network?

$$W = C\left(1 - e^{-0.12h}\right)$$

6. A construction crew has been assigned to build an apartment complex. The work of the crew can be modeled using the exponential formula below, where A is the total number of apartments to be built, w is the number of weeks, and F is the number of finished apartments. Out of a total of 100 apartments, how many apartments have been finished after 4 weeks of work?

$$F = A\left(1 - e^{-0.1w}\right)$$

7. The half-life of radium is approximately 1600 years.

a. Determine a so that $A(t) = A_0 a^t$ describes the amount of radium left after t years, where A_0 is the amount at time $t = 0$.

b. How much of a 1-gram sample of radium would remain after 100 years?

c. How much of a 1-gram sample of radium would remain after 1000 years?

8. The radioactive element polonium-210 has a relatively short half-life of 138 days, and one way to model the amount of polonium-210 remaining after t days is with the function $A(t) = A_0 e^{-0.005023t}$, where A_0 is the mass at time $t = 0$ (note that $A(138) = \frac{A_0}{2}$.) What percentage of the original mass of a sample of polonium-210 remains after 365 days?

9. A certain species of fish is to be introduced into a new man-made lake, and wildlife experts estimate the population will grow according to $P(t) = (1000)2^{\frac{t}{3}}$, where t represents the number of years from the time of introduction.

a. What is the doubling time for this population of fish?

b. How long will it take for the population to reach 8000 fish, according to this model?

10. The population of a certain inner-city area is estimated to be declining according to the model $P(t) = 237{,}000e^{-0.018t}$, where t is the number of years from the present. What does this model predict the population will be in ten years?

11. In an effort to control vegetation overgrowth, 100 rabbits are released in an isolated area that is free of predators. After one year, it is estimated that the rabbit population has increased to 500. Assuming exponential population growth, what will the population be after another six months?

12. Assuming a current world population of 7.75 billion people, an annual growth rate of 1.9% per year, and a worst-case scenario of exponential growth, what will the world population be in **a.** 10 years? **b.** 50 years?

13. Madiha has \$3500 that she wants to invest in a simple savings account for two and a half years, at which time she plans to close out the account and use the money as a down payment on a car. She finds one local bank offering an annual interest rate of 2.75% compounded monthly, and another bank offering an annual interest rate of 2.7% compounded daily (365 times per year). Which bank should she choose?

14. Madiha, from the last problem, does some more searching and finds an online bank offering an annual rate of 2.75% compounded continuously. How much more money will she earn over two and a half years if she chooses this bank rather than the local bank offering the same rate compounded monthly?

15. Tom hopes to earn \$1000 in interest in three years time from \$10,000 that he has available to invest. To decide if it's feasible to do this by investing in a simple monthly compounded savings account, he needs to determine the annual interest rate such an account would have to offer for him to meet his goal. What would the annual rate of interest have to be?

16. An investment firm claims that its clients usually double their principal in five years time. What annual rate of interest would a savings account, compounded monthly, have to offer in order to match this claim?

17. The function $C(t) = C_0(1+r)^t$ models the rise in the cost of a product that has a cost of C_0 today, subject to an average yearly inflation rate of r for t years. If the average annual rate of inflation over the next decade is assumed to be 3%, what will the inflation-adjusted cost of a \$100,000 house be in 10 years? Round your answer to the nearest dollar.

18. Given the inflation model $C(t) = C_0(1+r)^t$ (see Exercise 17), and given that a loaf of bread that currently sells for \$3.60 sold for \$3.10 six years ago, what has the average annual rate of inflation been for the past six years?

19. The function $N(t) = \dfrac{10,000}{1+999e^{-t}}$ models the number of people in a small town who have caught the flu t weeks after the initial outbreak.

a. How many people were ill initially?
b. How many people have caught the flu after eight weeks?
c. Determine what happens to the function $N(t)$ as $t \to \infty$.

20. The concentration $C(t)$, in milligrams per liter, of a certain drug in the bloodstream after t minutes is given by the formula $C(t) = 0.05(1 - e^{-0.2t})$. What is the concentration after 10 minutes?

21. Carbon-11 has a radioactive half-life of approximately 20 minutes, and is useful as a diagnostic tool in certain medical applications. Because of the relatively short half-life, time is a crucial factor when conducting experiments with this element.
a. Determine a so that $A(t) = A_0a^t$ describes the amount of carbon-11 left after t minutes, where A_0 is the amount at time $t = 0$.
b. How much of a 2 kg sample of carbon-11 would be left after 30 minutes?
c. How many milligrams of a 2 kg sample of carbon-11 would be left after six hours?

22. Charles has recently inherited \$8000 that he wants to deposit into a savings account. He has determined that his two best bets are an account that compounds annually at a rate of 3.20% and an account that compounds continuously at an annual rate of 3.15%. Which account would pay Charles more interest?

23. Marshall invests $1250 in a mutual fund which boasts a 5.7% annual return compounded semiannually (twice a year). After three and a half years, Marshall decides to withdraw his money.

a. How much is in his account?

b. How much has he made in interest from his investment?

24. Adam is working in a lab testing bacteria populations. After starting out with a population of 375 bacteria, he observes the change in population and notices that the population doubles every 27 minutes.

a. Find the equation for the population P in terms of time t in minutes, rounding a to the nearest thousandth.

b. Find the population after two hours.

25. Your credit union offers a special interest rate of 10% compounded monthly for the first year for a student savings account opened in August if the student deposits $5000 or more. You received a total of $9000 for graduation, and you decide to deposit all of it in this special account. Assuming you open your account in August and make no withdrawals for the first year, how much money will you have in your account at the end of February (after six months)? How much will you have at the end of the following July (after one full year)?

26. You have a savings account of $3000 with an interest rate of 6.8%.

a. How much interest would be earned in two years if the interest is compounded annually?

b. How much interest would be earned in two years if the interest is compounded semiannually?

c. In which case do you make more money on interest? Explain why this is so.

27. If $2500 is invested in a continuously compounded certificate of deposit with an annual interest rate of 4.2%, what would be the account balance at the end of three years?

28. The new furniture store in town boasts a special in which you can buy any set of furniture in their store and make no monthly payments for the first year. However, the fine print says that the interest rate of 7.25% is compounded quarterly beginning when you buy the furniture. You are considering buying a set of living room furniture for $4000 but know you cannot save up more than $4500 in one year's time. Can you fully pay off your furniture on the one year anniversary of having bought the furniture? If so, how much money will you have left over? If not, how much more money will you need?

29. When Nicole was born, her grandmother was so excited about her birth that she opened a certificate of deposit in Nicole's honor to help send her to college. Now at age 18, Nicole's account has $81,262.93. How much did her grandmother originally invest if the interest rate has been 8.1% compounded annually?

30. Inflation is a relative measure of your purchasing power over time. The formula for inflation is the same as the compound interest formula, but with $n = 1$. Given the current values below, what will the values of the following items be 10 years from now if inflation is at 6.4%?

a. an SUV: $38,000

b. a loaf of bread: $1.79

c. a gallon of milk: $3.40

d. your salary: $34,000

31. Depreciation is the decrease of an item's value and can be determined using a formula similar to that for compound interest: $V = P(1-r)^t$, where V is the new value. If the particular car you buy upon graduation from college costs \$17,500 and depreciates at a rate of 16% per year, what will the value of the car be in 5 years when you pay it off?

32. Assume the interest on your credit card is compounded continuously with an APR (annual percentage rate) of 19.8%. If you put your first term bill of \$3984 on your credit card, but do not have to make payments until you graduate (4 years later), how much will you owe when you start making payments?

33. Suppose you deposit \$5000 in an account for five years at an annual interest rate of 8.5%.

a. What would be the ending account balance if the interest is continuously compounded?

b. What would be the ending account balance if the interest is compounded daily?

c. Are these two answers similar? Why or why not?

TECHNOLOGY

Use a graphing utility to sketch the graphs of the following functions.

34. $m(x) = 1 - 3e^x$

35. $p(x) = e^{4x} - 2$

36. $b(x) = \dfrac{1}{e^{x-2}}$

37. $m(x) = e^{2x^2-3x+1}$

38. $g(x) = e^{x+3} - 3$

39. $m(x) = 6e^{2x} - 2$