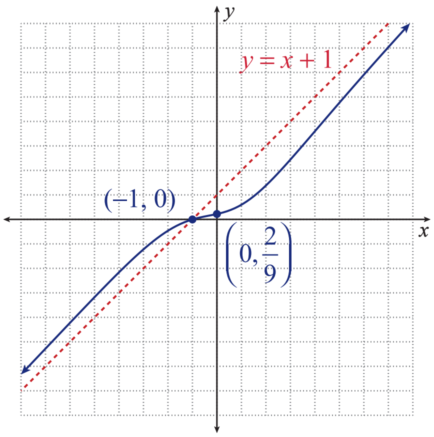


c. The denominator of this function cannot be factored, so there are no restrictions on the domain of h. Further, we saw in Example 2b that this function has an oblique asymptote of $y = x + 1$.

As usual, we calculate the y-intercept by substituting $x = 0$.

$$h(0) = \frac{0^3 + 0^2 + 2(0) + 2}{0^2 + 9} = \frac{2}{9}$$

There are different approaches to finding the x-intercepts. Looking at the numerator, we might guess that -1 is a zero of the numerator. A quick calculation confirms this: $(-1)^3 + (-1)^2 + 2(-1) + 2 = 0$. This means we can factor the numerator. Using division, we find $x^3 + x^2 + 2x + 2 = (x+1)(x^2+2)$. Thus, $(-1, 0)$ is the only x-intercept.

With the intercepts and a few other plotted points, we obtain the graph of h.

3.8 EXERCISES

PRACTICE

Find equations for the vertical asymptotes, if any, for each of the following rational functions. See Example 1.

1. $f(x) = \dfrac{5}{x-1}$

2. $f(x) = \dfrac{x^2+3}{x+3}$

3. $f(x) = \dfrac{x^2-4}{x+2}$

4. $f(x) = \dfrac{-3x+5}{x-2}$

5. $f(x) = \dfrac{3x^2+1}{x-2}$

6. $f(x) = \dfrac{x^2+2x}{x+1}$

7. $f(x) = \dfrac{x^2-4}{2x-x^2}$

8. $f(x) = \dfrac{x+2}{x^2-9}$

9. $f(x) = \dfrac{x^2-2x-3}{2x^2-5x-3}$

10. $f(x) = \dfrac{2x^2+2x-4}{x^2+2x+1}$

11. $f(x) = \dfrac{x^3-27}{x^2+5}$

12. $f(x) = \dfrac{x^2+5}{x^3-27}$

13. $f(x) = \dfrac{x^2-1}{x^2-8x+7}$

14. $f(x) = \dfrac{2x^2+7x-14}{2x^2+7x-15}$

15. $f(x) = \dfrac{x^3-6x^2+11x-6}{x^3+8}$

16. $f(x) = \dfrac{x^2-2x-15}{x-5}$

17. $f(x) = \dfrac{x^2-16}{x^2-4}$

18. $f(x) = \dfrac{x^2+4x+4}{x^2+x-2}$

Find equations for the horizontal or oblique asymptotes, if any, for each of the following rational functions. See Example 2.

19. $f(x)=\dfrac{5}{x-1}$

20. $f(x)=\dfrac{x^2+3}{x+3}$

21. $f(x)=\dfrac{x^4-4}{x^2+2}$

22. $f(x)=\dfrac{x^2-4}{2x-x^2}$

23. $f(x)=\dfrac{x+2}{x^2-9}$

24. $f(x)=\dfrac{x^2-2x-3}{2x^2-5x-3}$

25. $f(x)=\dfrac{2x^2+2x-4}{x^2+2x+1}$

26. $f(x)=\dfrac{-3x+5}{x-2}$

27. $f(x)=\dfrac{3x^2+1}{x-2}$

28. $f(x)=\dfrac{x^3-27}{x^2+5}$

29. $f(x)=\dfrac{x^2+5}{x^3-27}$

30. $f(x)=\dfrac{x^2+2x}{x+1}$

31. $f(x)=\dfrac{x^2-81}{x^3+7x-12}$

32. $f(x)=\dfrac{x^3-3x^2+2x}{x-7}$

33. $f(x)=\dfrac{x^2-9x+4}{x+2}$

34. $f(x)=\dfrac{-x^5+2x^2}{5x^5+3x^3-7}$

35. $f(x)=\dfrac{5x^2-x+12}{x-1}$

36. $f(x)=\dfrac{2x^2-5x+6}{x-3}$

Sketch the graphs of the following rational functions, making use of your work in the previous exercises and additional information about intercepts and any other points that may be useful. See Example 3.

37. $f(x)=\dfrac{5}{x-1}$

38. $f(x)=\dfrac{x^2+3}{x+3}$

39. $f(x)=\dfrac{x^2-4}{x+2}$

40. $f(x)=\dfrac{x^2-4}{2x-x^2}$

41. $f(x)=\dfrac{x+2}{x^2-9}$

42. $f(x)=\dfrac{x^2-2x-3}{2x^2-5x-3}$

43. $f(x)=\dfrac{2x^2+2x-4}{x^2+2x+1}$

44. $f(x)=\dfrac{-3x+5}{x-2}$

45. $f(x)=\dfrac{3x^2+1}{x-2}$

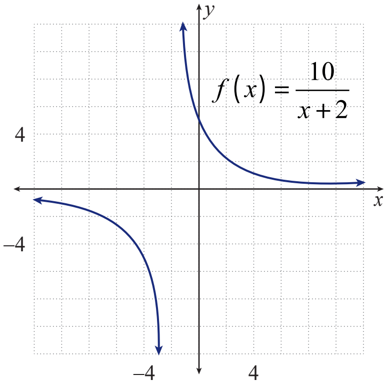
46. $f(x)=\dfrac{x^3-27}{x^2+5}$

47. $f(x)=\dfrac{x^2+5}{x^3-27}$

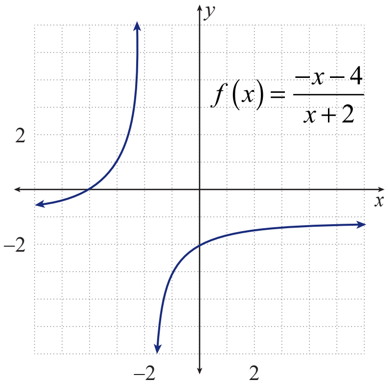
48. $f(x)=\dfrac{x^2+2x}{x+1}$

For each graph, find any **a.** vertical asymptotes, **b.** horizontal asymptotes, **c.** oblique asymptotes, **d.** visible *x*-intercepts, or **e.** visible *y*-intercepts.

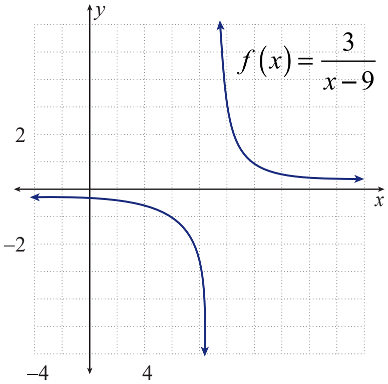
49.



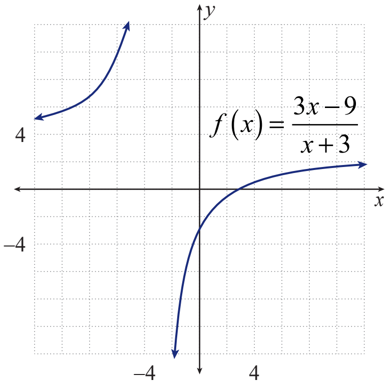
50.



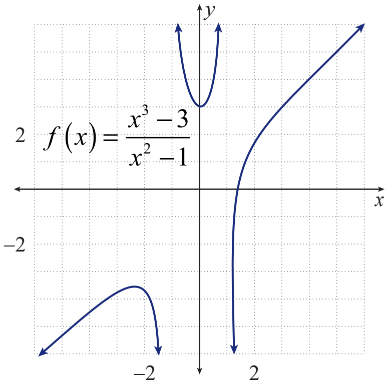
51.



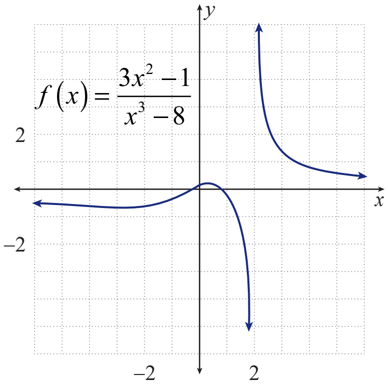
52.



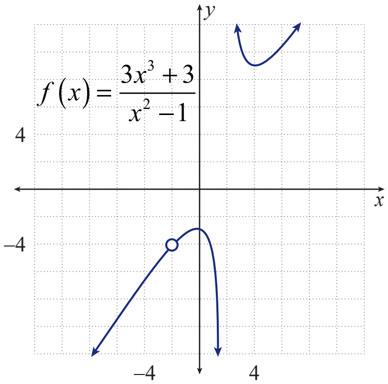
53.



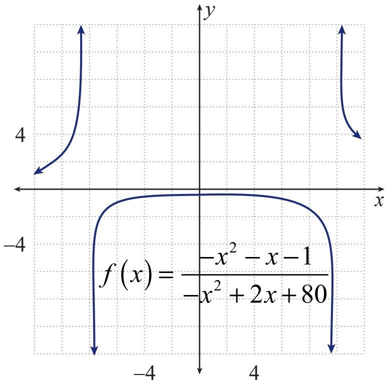
54.



55.



56.



APPLICATIONS

57. April raises a species of aquarium fish, and the total number of fish she has follows the formula

$$p(t) = \frac{200t}{t+1},$$

where $t \geq 0$ represents the number of months since she began.

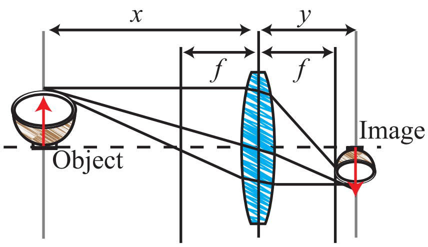
a. Sketch the graph of $p(t)$ for $t \geq 0$.

b. What happens to April's fish population in the long run?

58. If an object is placed a distance x from a lens with a focal length of f, the image of the object will appear a distance y on the opposite side of the lens, where x, f, and y are related by the equation $\frac{1}{x} + \frac{1}{y} = \frac{1}{f}$.

a. Express y as a function of x and f.

b. Graph your function for a lens with a focal length of 30 mm $(f = 30)$. What happens to y as the distance x increases?



59. At t minutes after injection, the concentration (in mg/L) of a certain drug in the bloodstream of a patient is given by the formula

$$c(t) = \frac{20t}{t^2+1}.$$

a. Sketch the graph of $c(t)$ for $t \geq 0$.

b. What happens to the concentration of the drug in the long run?