

3.6 EXERCISES

💡 PRACTICE

For each function or graph, determine the basic function that has been shifted, reflected, stretched, or compressed.

1. $f(x) = -(1-x)^2 + 2$

2. $f(x) = \frac{1}{x-4} + 5$

3. $f(x) = \sqrt[3]{x+6} - 2$

4. $f(x) = -2 + 2|x-3|$

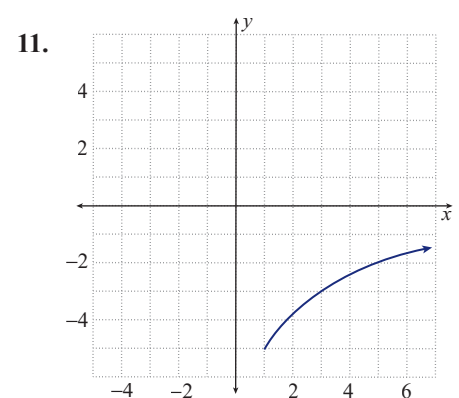
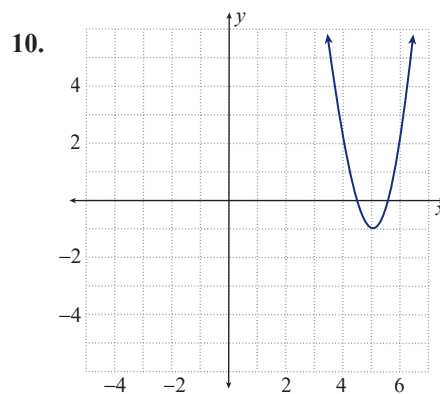
5. $f(x) = \sqrt{x+2} - 5$

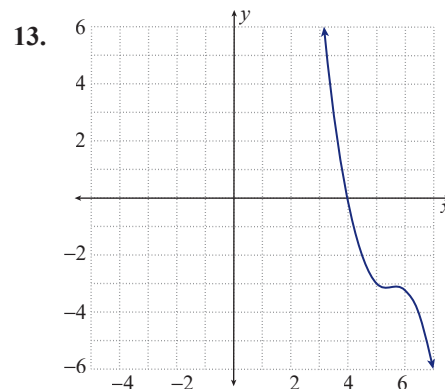
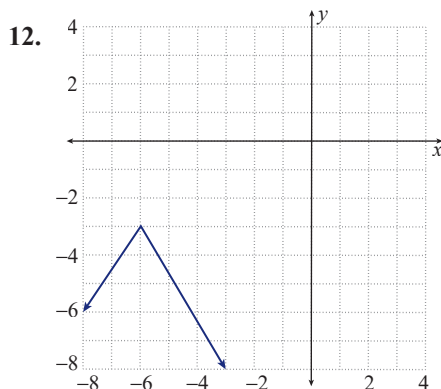
6. $f(x) = \lfloor -2 - x \rfloor$

7. $f(x) = \frac{1}{(x+2)^2} + 1$

8. $f(x) = \frac{\sqrt{-x}}{2} + 4$

9. $f(x) = (x+6)^3$





Sketch the graphs of the following functions by first identifying the more basic functions that have been shifted, reflected, stretched, or compressed. Then determine the domain and range of each function. See Examples 1 through 6.

14. $f(x) = (x+2)^3$

15. $G(x) = |x-4|$

16. $p(x) = -(x+1)^2 + 2$

17. $g(x) = \sqrt{x+3} - 1$

18. $q(x) = (1-x)^2$

19. $r(x) = -\sqrt[3]{x}$

20. $s(x) = \sqrt{2-x}$

21. $F(x) = \frac{|x+2|}{3} + 3$

22. $w(x) = \frac{1}{(x-3)^2}$

23. $v(x) = \frac{1}{3x} - 2$

24. $f(x) = \frac{1}{2-x}$

25. $k(x) = \sqrt{-x} + 2$

26. $b(x) = \sqrt[3]{x+2} - 5$

27. $b(x) = \llbracket x-4 \rrbracket + 4$

28. $R(x) = 4 - 2|x|$

29. $S(x) = (3-x)^3$

30. $g(x) = -\frac{1}{x+1}$

31. $h(x) = \frac{x^2}{2} - 3$

32. $W(x) = 1 - |4-x|$

33. $W(x) = -\frac{|x-1|}{4}$

34. $S(x) = \frac{1}{x^2} + 3$

35. $V(x) = -3\sqrt{x-1} + 2$

36. $g(x) = x^2 - 6x + 9$ (**Hint:** Find a better way to write the function.)

37. $h(x) = \frac{|x|}{x}$ (**Hint:** Evaluate h at a few points to understand its behavior.)

38. $W(x) = \frac{x-1}{|x-1|}$

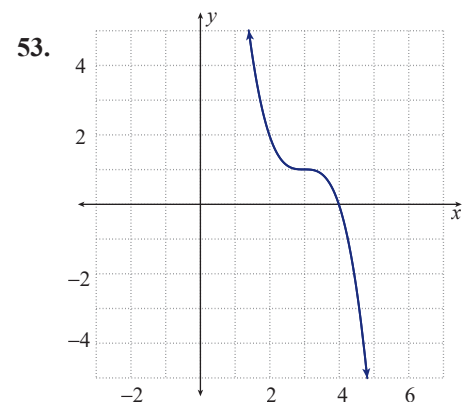
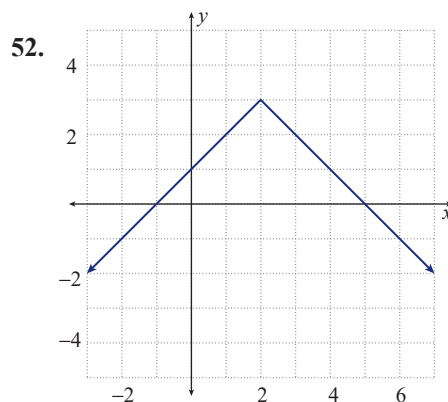
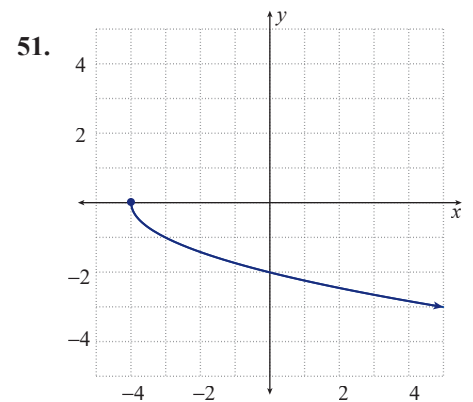
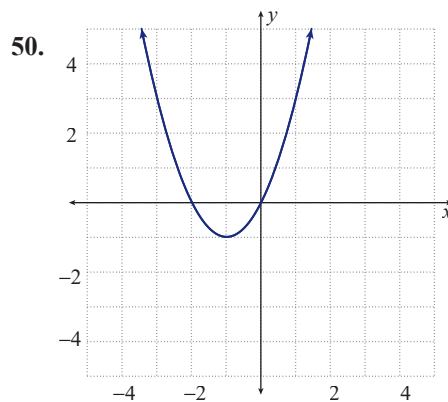
39. $s(x) = \llbracket x-2 \rrbracket$

Write a formula for each of the functions described.

40. Use the function $g(x) = x^2$. Move the function 3 units to the left and 4 units down.

41. Use the function $g(x) = x^2$. Move the function 4 units to the right and 2 units up.
42. Use the function $g(x) = x^2$. Reflect the function across the x -axis and move it 6 units up.
43. Use the function $g(x) = x^2$. Move the function 2 units to the right and reflect across the y -axis.
44. Use the function $g(x) = x^3$. Move the function 1 unit to the left and reflect across the y -axis.
45. Use the function $g(x) = x^3$. Move the function 10 units to the right and 4 units up.
46. Use the function $g(x) = \sqrt{x}$. Move the function 5 units to the left and reflect across the x -axis.
47. Use the function $g(x) = \sqrt{x}$. Reflect the function across the y -axis and move it 3 units down.
48. Use the function $g(x) = |x|$. Move the function 7 units to the left, reflect across the x -axis, and reflect across the y -axis.
49. Use the function $g(x) = |x|$. Move the function 8 units to the right, 2 units up, and reflect across the x -axis.

Use your knowledge about transformations to find a possible formula for the function $f(x)$ given its graph.



Determine if each of the following relations is a function. If so, determine whether it is even, odd, or neither. Also determine if it has y -axis symmetry, x -axis symmetry, origin symmetry, or none of these symmetries, and then sketch the graph of the relation. See Example 7.

54. $f(x) = |x| + 3$

55. $g(x) = x^3$

56. $h(x) = x^3 - 1$

57. $w(x) = \sqrt[3]{x}$

58. $x = -y^2$

59. $3y - 2x = 1$

60. $x + y = 1$

61. $F(x) = (x-1)^2$

62. $x = y^2 + 1$

63. $x = 2|y|$

64. $g(x) = \frac{x^2}{5} - 5$

65. $s(x) = \left\lfloor x + \frac{1}{2} \right\rfloor$

66. $m(x) = \sqrt[3]{x} - 1$

67. $xy = 2$

68. $x + y^2 = 3$

For each of the following functions, find the open intervals of monotonicity where the function is increasing, decreasing, or constant. See Examples 8 and 9.

69. $f(x) = (x+3)^2$

70. $g(x) = -|x-2|$

71. $h(x) = \frac{1}{x-1}$

72. $H(x) = \frac{1}{(x+3)^2}$

73. $G(x) = \sqrt{x+1}$

74. $F(x) = -2$

75. $p(x) = -30|x-1|$

76. $q(x) = (4-x)^2 + 1$

77. $r(x) = \frac{(x-7)^4}{-2} + 4$

78. $P(x) = \begin{cases} (x+3)^2 & \text{if } x < -1 \\ 1 & \text{if } x \geq -1 \end{cases}$

79. $Q(x) = \begin{cases} |x-1| & \text{if } x \leq 3 \\ 5-x & \text{if } x > 3 \end{cases}$

APPLICATIONS

80. During the summer months, the water level of a garden pool varies as water is added and as it evaporates. On May 1st the pool was 3.4 feet deep. After a steady and linear increase due to rain, the depth had increased to 4.9 feet on June 1st. By July 1st the water level had decreased linearly to 4.2 feet. Knowing that the pool would be covered for the winter, the owner filled the pool (in an essentially linear fashion) until it reached 5 feet on August 1st. Graph the water level as a function of time and determine the open intervals of monotonicity.

81. The profit made by a hot dog vendor is given by the function

$$P(x) = \begin{cases} 2x - 3 & \text{if } x \geq 0 \text{ and } x < 7 \\ \frac{1}{4}x^2 & \text{if } x \geq 7 \end{cases}$$

where x is the number of hot dogs sold. Graph the profit function and determine the open intervals of monotonicity.

82. The cost incurred by a newspaper stand is given by the function

$$C(x) = \begin{cases} -2\sqrt{x} + 8 & \text{if } x \geq 0 \text{ and } x < 3 \\ -x + 8 & \text{if } x \geq 3 \end{cases}$$

where x is the number of newspapers sold. Graph the cost function and determine the open intervals of monotonicity.

 TECHNOLOGY

Mentally sketch the graph of the given function by identifying the basic shape that has been shifted, reflected, stretched, or compressed. Then use a graphing utility to graph the function and check your reasoning.

83. $f(x) = -2(3-x)^3 + 5$

84. $f(x) = \frac{3}{x+5} - 1$

85. $f(x) = \frac{-1}{(x-2)^2} - 3$

86. $f(x) = -3|x+2| - 4$

87. $f(x) = -\sqrt{1-x} + 2$

88. $f(x) = \sqrt[3]{2+x} - 1$

Write a possible equation for the function depicted on the graphing utility. The function is shown in a $[-10,10]$ by $[-10,10]$ viewing window.

