16.4 EXERCISES

PRACTICE

In Exercises 1–8, use the method of Lagrange multipliers to find the minimum value of f subject to the given constraint.

1. $f(x,y) = x^2 + y^2$, subject to $x + y - 4 = 0$

2. $f(x,y) = 4x^2 + 3y^2$, subject to $x + y - 7 = 0$

3. $f(x,y) = 5x^2 + 4y^2 - 2x$, subject to $x - y - 2 = 0$

4. $f(x,y) = 2x^2 + y^2 - 18x$, subject to $3x - y - 8 = 0$

5. $f(x,y) = 6x^2 + 5y^2 - xy$, subject to $2x + y = 24$

6. $f(x,y) = 2x^2 + 3y^2 - 3xy$, subject to $x + y = 16$

7. $f(x,y) = x^3 + y^3$, subject to $x + y = 8$

8. $f(x,y) = x^3 - y^3$, subject to $x - y = 10$

In Exercises 9–16, use the method of Lagrange multipliers to find the maximum value of f subject to the given constraint.

9. $f(x,y) = 2x^2 - 5y^2$, subject to $x - y = 3$

10. $f(x,y) = 5y^2 - 8x^2$, subject to $x + y = 6$

11. $f(x,y) = 8xy - 3x^2$, subject to $x + 2y = 14$

12. $f(x,y) = 6x^2 - 5xy$, subject to $2x - y = 8$

13. $f(x,y) = x^2 + y^2 + 4xy$, subject to $3x + 4y = 23$

14. $f(x,y) = x^2 - 4y^2 + 84xy$, subject to $5x + 2y = 18$

15. $f(x,y) = 15x^{0.4}y^{0.6}$, subject to $10x + 8y = 200$

16. $f(x,y) = 8x^{\frac{1}{2}}y^{\frac{1}{2}}$, subject to $6x + 15y = 450$

In Exercises 17–20, use the method of Lagrange multipliers to find the maximum and minimum values of f subject to the given constraint.

17. $f(x, y) = 4xy$, subject to $x^2 + 4y^2 = 72$

18. $f(x, y) = 5xy$, subject to $9x^2 + y^2 = 162$

19. $f(x,y) = x^3 + 4y^3$, subject to $x + y = 6$

20. $f(x,y) = 3x^3 + y^3$, subject to $3x + y = 8$

APPLICATIONS

21. Cost: A company has a plant in Los Angeles and a plant in Oklahoma City. The firm is committed to produce a total of 40 units of a product each week. The cost function is given by $C(x, y) = 0.3x^2 + 0.2y^2 + 20x + 7y + 200,$ where x is the number of units produced in Los Angeles and y is the number of units produced in Oklahoma City. How many units should be produced in each plant to minimize the total weekly costs?

22. Profit: A department store sells two styles of a jacket, lined and unlined. During the month of January, the management expects to sell exactly 250 jackets. The profit function is given by $P(x, y) = -0.3x^2 - 0.4y^2 - 0.3xy + 80x + 65y - 1000,$ where x is the number of lined jackets sold and y is the number of unlined jackets sold. How many of each type should be sold to maximize the profit?

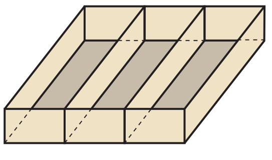
23. Production: The production function for a certain product is given by $f(x, y) = 75x^{0.3}y^{0.7},$ where x is the number of units of labor and y is the number of units of capital. Each unit of labor costs \$300, and each unit of capital costs \$200. If the company's budget allows a total of \$20,000 for labor and capital, find the maximum level of production.

24. Production: The management of a company has determined that x units of labor and y units of capital are required to produce $f(x, y) = 130x^{0.4}y^{0.6}$ units of a product. Each unit of labor costs \$450, and each unit of capital costs \$360. Find the maximum number of units that can be produced if a total of \$90,000 is available for labor and capital.

25. Sales: A sales representative for a textbook publishing company estimates her monthly sales for March to be $S(x, y) = 30x + 18y - 1.2x^2 - 0.6y^2$ in thousands of dollars, where x and y represent the number of days spent in each of the two metropolitan areas that comprise her sales territory. If she plans to work 20 days during the month, how many days should she spend in each area to maximize her sales?

26. Revenue: The marketing manager of a department store has determined that revenue, in dollars, is related to the number of units of television advertising x and the number of units of newspaper advertising y by the function $R(x, y) = 500(20x + 5y + 6xy - x^2).$ Each unit of television advertising costs \$3000, and each unit of newspaper advertising costs \$1500. If the advertising budget is \$30,000, find the maximum revenue.

27. Construction: A farmer wants to build a rectangular pen and then divide it with two interior fences. The total area enclosed is to be 2484 ft^2. The exterior fence costs \$18 per foot, and the interior fence costs \$16.50 per foot. Find the dimensions of the pen that will minimize the cost of fencing.



28. Shipping: A container manufacturer is asked to design a closed rectangular shipping crate with a square base. The volume of the crate is 36 ft^3. The material for the top costs \$1 per square foot, the material for the sides costs \$0.90 per square foot, and the material for the bottom costs \$1.40 per square foot. Find the dimensions that will minimize the total cost of building the crate.