c. Treat x and y as constants. In this case, the entire expression $2ye^{xy}$ is treated as a constant.

$$\frac{\partial w}{\partial z} = 0 + 2z = 2z$$

16.2 EXERCISES

PRACTICE

Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for each of the functions in Exercises 1–28.

1. $f(x,y) = 4x + 7y - 10$

2. $f(x,y) = 11x - 19y + 2$

3. $f(x,y) = 2x^2 + 5y^2$

4. $f(x,y) = 5x^3 - 6y^4$

5. $f(x,y) = x^2y + 4xy^3 + 6$

6. $f(x,y) = x^3 - 90x^2y^3 - 9$

7. $f(x,y) = y\sqrt{25 + x^2}$

8. $f(x,y) = x^3\sqrt{y^2 + 5}$

9. $f(x,y) = \sqrt{49 - x^2 - y^2}$

10. $f(x,y) = \sqrt{16 + 2x^2 + y^2}$

11. $f(x,y) = 4e^{x-y}$

12. $f(x,y) = 7e^{xy}$

13. $f(x,y) = x \ln y$

14. $f(x,y) = \ln(x^2 + y^2)$

15. $f(x,y) = \ln(x^2 + 3xy)$

16. $f(x,y) = \dfrac{y}{\ln x}$

17. $f(x,y) = \dfrac{2x^2}{y^2 + 1}$

18. $f(x,y) = \dfrac{x^2 + 3}{5y - 9}$

19. $f(x,y) = \dfrac{x^2}{xy + 3}$

20. $f(x,y) = \dfrac{x + y}{4x - y}$

21. $f(x,y) = x^3e^y + ye^{x^2}$

22. $f(x,y) = xe^{y^3} - y^2e^x$

23. $f(x,y) = x\sqrt{xy + 2}$

24. $f(x,y) = y\sqrt{x^2 + y^3}$

25. $f(x,y) = x^4e^{xy}$

26. $f(x,y) = y^3e^{-xy}$

27. $f(x,y) = y^5 \ln(x^2 - 5y^2)$

28. $f(x,y) = 3x \ln xy^4$

In Exercises 29–32, find $\frac{\partial S}{\partial m}$ and $\frac{\partial S}{\partial b}$.

29. $S(m,b) = (2m + b - 9)^2 + (4m + b - 13)^2 + (5m + b - 18)^2$

30. $S(m,b) = (8m + b - 17)^2 + (9m + b - 23)^2 + (10m + b - 28)^2$

31. $S(m,b) = (6m + b - 40)^2 + (8m + b - 49)^2 + (9m + b - 55)^2 + (10m + b - 62)^2$

32. $S(m,b) = (12m + b - 81)^2 + (13m + b - 88)^2 + (14m + b - 96)^2 + (15m + b - 101)^2$

In Exercises 33–44, find all second-order partial derivatives f_{xx}, f_{xy}, f_{yx}, and f_{yy}.

33. $f(x,y) = 3xy + x^2y^3 - 19$

34. $f(x,y) = 5x^3y - 3x^3y^2 - 13$

35. $f(x,y) = x^4y^{\frac{2}{3}}$

36. $f(x,y) = (xy)^{\frac{3}{4}}$

37. $f(x,y) = xe^{2y}$

38. $f(x,y) = \dfrac{e^{x^3}}{y^4}$

39. $f(x,y) = (4x - 3y)^{\frac{5}{3}}$

40. $f(x,y) = \sqrt{7x^3 + y^2}$

41. $f(x,y) = \dfrac{3x+1}{5y+3}$

42. $f(x,y) = \dfrac{6y^2 - 5}{2x+7}$

43. $f(x,y) = \dfrac{2xy}{x-y}$

44. $f(x,y) = \dfrac{x-y}{xy}$

In Exercises 45–48, find f_x, f_y, and f_z.

45. $f(x,y,z) = xy + 2xz + 9yz$

46. $f(x,y,z) = 3x^2y + 2xyz + 7xz^2$

47. $f(x,y,z) = \left(8x^2 + 5y^2 - 2z^2\right)^2$

48. $f(x,y,z) = \sqrt{x^2 + 2y^2 + 4z^2}$

In Exercises 49–52, find $\dfrac{\partial F}{\partial x}, \dfrac{\partial F}{\partial y}$, and $\dfrac{\partial F}{\partial \lambda}$. (Note that λ is the Greek letter lambda.)

49. $F(x,y,\lambda) = 8x + 15xy - 2y^2 + \lambda(x + y - 60)$

50. $F(x,y,\lambda) = 3x^2 + 12y^2 + \lambda(x + 2y - 84)$

51. $F(x,y,\lambda) = 5x^2 + 3xy - 10y^2 + \lambda(14x + 17y - 49)$

52. $F(x,y,\lambda) = 7x^2 - 2xy + 9y^2 + \lambda(8x + 15y - 120)$

APPLICATIONS

53. Marginal productivity: The number of units of a product that are manufactured by a company is given by $f(L,K) = 80L^{\frac{2}{3}}K^{\frac{1}{3}}$, where L is the units of labor and K is the units of capital. Find the marginal productivity of labor and the marginal productivity of capital if the company is currently utilizing 27 units of labor and 64 units of capital.

54. Marginal productivity: The productivity of a company is approximated by $f(L,K) = 20L^{\frac{2}{5}}K^{\frac{3}{5}}$, where L is the units of labor and K is the units of capital. Find the marginal productivity of labor and the marginal productivity of capital if the company is currently utilizing 32 units of labor and 32 units of capital.

55. Marginal cost: A company manufactures two products, product A and product B. The cost of producing x units of A and y units of B is $C(x,y) = 3000 + 7x + 5.8y + 0.03x^2 - xy + 0.02y^2$.

a. Find the marginal cost with respect to x.

b. Find the marginal cost with respect to y.

56. Marginal profit: The profit from the sale of two products is given by the function $P(x, y) = 88x + 54y - 0.02x^2 - 0.015y^2 - 68$, where x is the number of units of product A sold, and y is the number of units of product B sold.

a. Find the marginal profit with respect to x.

b. Find the marginal profit with respect to y.

57. Marginal profit: A company produces two models of a product. The cost function is given by $C(x, y) = x^2 - 2xy + 2y^2 + 4x + 3y + 8$ and the revenue function is given by $R(x, y) = 20x + 15y$, where x is the number of units of model A and y is the number of units of model B produced and sold.

a. Find the profit function.

b. Find $P_x(20,14)$ and $P_y(20,14)$ and interpret the results.

58. Marginal profit: A firm produces and sells x units of product A and y units of product B. Its revenue function is given by $R(x, y) = 80x + 100y$ and its cost function is given by $C(x, y) = x^2 + 1.5y^2 - xy + 1500$. Find $P_x(50, 25)$ and $P_y(50, 25)$ and interpret the results.

59. Marginal profit: A marketing manager of a department store has determined that revenue is related to the number of units of television advertising x and the number of units of newspaper advertising y by the function $R(x, y) = 500(20x + 5y + 20xy - x^2)$. Each unit of television advertising costs \$5000 and each unit of newspaper advertising costs \$2500.

a. Find the marginal profit with respect to x.

b. Find the marginal profit with respect to y.

60. Marginal profit: A firm manufactures and sells two models of electric mowers. The standard model of the mower sells for \$300, and the self-propelled model of the mower sells for \$400. The total cost function is $C(x, y) = 90{,}000 + 0.05x^2 + 0.1y^2 + 0.125xy$, where x is the number of standard models and y is the number of self-propelled models.

a. Find the marginal profit with respect to x.

b. Find the marginal profit with respect to y.