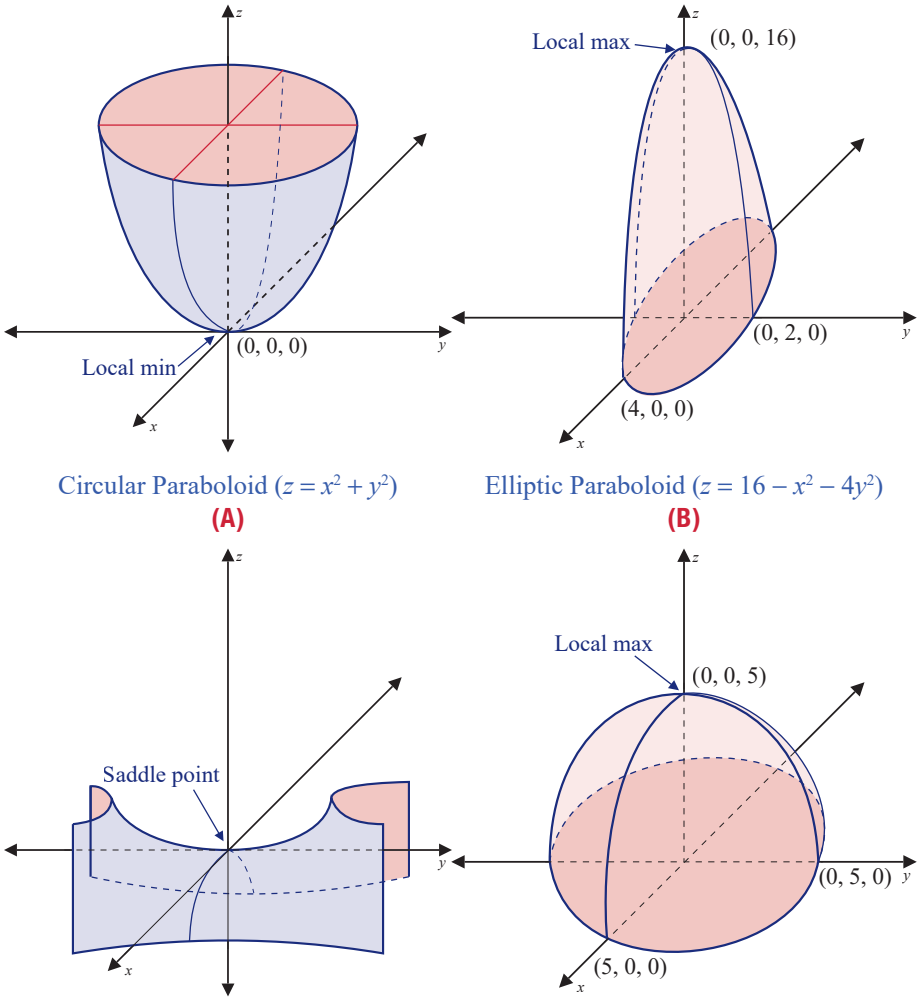


Circular Paraboloid ($z = x^2 + y^2$)
(A)

Elliptic Paraboloid ($z = 16 - x^2 - 4y^2$)
(B)

Hyperbolic Paraboloid ($z = y^2 - x^2$)
(C)

Hemisphere ($z = \sqrt{25 - x^2 - y^2}$)
(D)

FIGURE 4

16.1 EXERCISES

PRACTICE

In Exercises 1–15, find the indicated function values, if possible.

1. $f(x, y) = 12x - 3y + xy$
a. $f(1, 3)$
b. $f(0, 4)$

2. $f(x, y) = 7xy - 11x + 9y$
a. $f(-2, 1)$
b. $f(3, -2)$

3. $f(x, y) = 4x^2 - 3xy + y^2$
a. $f(2, 5)$
b. $f(0, 3)$

4. $g(x, y) = 2xy + 5x^2y + y^3$
a. $g(-2, 2)$
b. $g(-1, -1)$

5. $g(x, y) = \dfrac{4x + y}{x - y}$

a. $g(3, -1)$
b. $g(2, 2)$

6. $f(x, y) = \dfrac{2x - y}{x^2 + y}$

a. $f(6, 2)$
b. $f(3, -9)$

7. $g(x, y) = \dfrac{8x^2 y}{\sqrt{2x + y}}$

a. $g(1, -4)$
b. $g(-2, 6)$

8. $f(x, y) = \dfrac{5x^2 + y^3}{\sqrt{4x + y^2}}$

a. $f(-1, 3)$
b. $f(3, 2)$

9. $g(x, y) = 3xe^{x+y}$

a. $g(2, 1)$
b. $g(-1, 0)$

10. $g(x, y) = ye^{4x} + 2xy$

a. $g(1, 5)$
b. $g(-1, 3)$

11. $f(x, y) = x \ln xy + y \ln x$

a. $f(1, e)$
b. $f(e^2, 1)$

12. $f(x, y) = 2x^3 y + \ln y$

a. $f(-2, e)$
b. $f(3, 1)$

13. $A(P, r, t) = Pe^{rt}$

a. $A(1000, 0.08, 4)$
b. $A(500, 0.06, 5)$

14. $A(P, r, t) = P\left(1 + \dfrac{r}{4}\right)^{4t}$

a. $A(1500, 0.08, 6)$
b. $A(800, 0.06, 10)$

15. $S(l, w, h) = 2lw + 2lh + 2wh$

a. $S(18, 15, 9)$
b. $S(14, 9, 11)$

In Exercises 16–19, draw a three-dimensional coordinate system, and graph and label the given points.

16. $A(0, 0, 3);\ B(2, -1, 0);\ C(4, 2, 1);\ D(3, -2, -2)$

17. $A(0, 1, 0);\ B(3, 0, -4);\ C(1, 1, -3);\ D(-2, 3, 0)$

18. $A(-3, 0, 2);\ B(0, 0, 2);\ C(2, -2, 2);\ D(0, -2, 2)$

19. $A(-3, -2, 1);\ B(4, 0, -1);\ C(0, 0, 4);\ D(-1, 4, -1)$

In Exercises 20–25, draw a three-dimensional coordinate system, and graph the given planes.

20. $x = 0$ **21.** $y = 0$ **22.** $z = 0$

23. $x = 3$ **24.** $y = 2$ **25.** $z = 4$

APPLICATIONS

26. Stock yield: The yield of a stock is given by the function $Y(d, p) = \dfrac{d}{p}$, where d is the dividend per share of stock and p is the price per share. Find the yield of a stock that sells for \$5.88 if the dividend is \$1.00.

27. Intelligence quotient: The intelligence quotient (IQ) of a person is determined by $f(M, C) = 100 \cdot \frac{M}{C}$, where M is the mental age (determined by tests) and C is the actual or chronological age. Find the IQ of a child who is 13 years old and has a mental age of 15.4 years. (Round to the nearest integer.)

28. Cobb-Douglas production: The number of units of a product that are manufactured by a company is given by $f(L, K) = 300L^{0.4}K^{0.6}$, where L is the units of labor and K is the units of capital.

a. How many units of a product will be manufactured by utilizing 30 units of labor and 24 units of capital? (Round to the nearest unit.)

b. How many units will be produced if the number of units of labor and capital are doubled? (Round to the nearest unit.)

29. Cost: A company manufactures two lawn mower models, standard and self-propelled. The cost of producing each standard mower is \$80, and the cost of producing each self-propelled mower is \$140. If the fixed costs are \$5200, the total cost function is given by $C(x, y) = 5200 + 80x + 140y$, where x is the number of standard and y is the number of self-propelled mowers.

a. Find $C(30, 20)$.

b. Find $C(36, 25)$.

30. Cost: The cost function for producing two models of a product is found to be $C(x, y) = 850 + 32x + 20y$, where x is the number of model A and y is the number of model B. The cost for model A is \$32, the cost for model B is \$20, and the fixed costs are \$850 per week.

a. Find $C(40, 24)$.

b. Find $C(60, 38)$.

31. Cost: The cost of producing the standard model of a video camera is \$160. The cost of producing the deluxe model is \$220.

a. If a company has weekly fixed costs of \$1360, find the cost function $C(x, y)$, where x is the number of standard models and y is the number of deluxe models.

b. Find $C(15, 12)$.

32. Cost: A company makes two grades of paint, grade I, guaranteed for 5 years, and grade II, guaranteed for 10 years. A gallon of grade I costs \$3.20 to make, while a gallon of grade II costs \$3.90 to make. The weekly fixed costs are \$4500.

a. Find the cost function $C(x, y)$ for making x gallons of grade I and y gallons of grade II.

b. What is the cost of making 200 gallons of grade I and 140 gallons of grade II?

33. Revenue: A grocery store sells two brands of a product, the store brand and a name brand. The manager estimates that if she sells the store brand for x dollars and the name brand for y dollars, she will be able to sell $64 - 20x + 18y$ units of the store brand and $52 + 16x - 22y$ units of the name brand.

a. Find the revenue function $R(x, y)$.

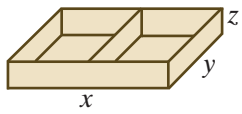
b. What is the revenue if she sells the store brand for \$4.00 and the name brand for \$4.50?

34. Revenue: A pharmacy sells two cold remedies, one a generic remedy and the other a name brand. The store manager has determined that he can sell $26 - 6x + 8y$ bottles of the generic remedy and $22 + 5x - 9y$ bottles of the name brand if the prices are x dollars per bottle and y dollars per bottle, respectively.

a. Find the revenue function $R(x, y)$.

b. What is the revenue if the generic remedy is priced at \$6.20 per bottle and the name brand is priced at \$7.00 per bottle?

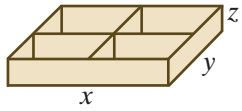
35. Volume and surface area: A rectangular box has no top and one partition (see diagram).



a. Write a function of three variables for the number of cubic units in the volume of the box.

b. Write a function of three variables for the number of square units of material needed to construct the box.

36. Volume and surface area: A rectangular box has no top and two intersecting partitions (see diagram).



a. Write a function of three variables for the number of cubic units in the volume of the box.

b. Write a function of three variables for the number of square units of material needed to construct the box.

37. Compound interest: A deposit of \$1000 is made into a savings account earning interest compounded quarterly. The amount $A(r, t)$ after t years is given by $A(r,t) = 1000\left(1 + \frac{r}{4}\right)^{4t}$, where r is the interest rate in decimal form. Use this function of two variables to complete the following table.

		Number of Years (t)		
		3	5	10
Rate (r)	0.06			
	0.08			
	0.10			

38. Interest compounded continuously: A deposit of \$1000 is made into a savings account earning interest compounded continuously. The amount $A(r, t)$ after t years is given by $A(r, t) = 1000e^{rt}$, where r is the interest rate in decimal form. Use this function of two variables to complete the following table.

		Number of Years (t)		
		5	8	12
Rate (r)	0.080			
	0.085			
	0.100			