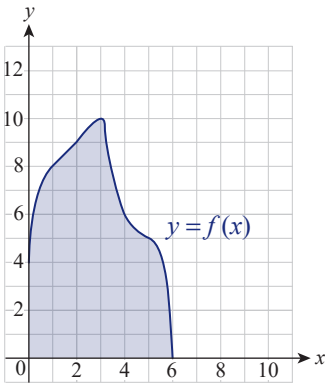
15.4 EXERCISES

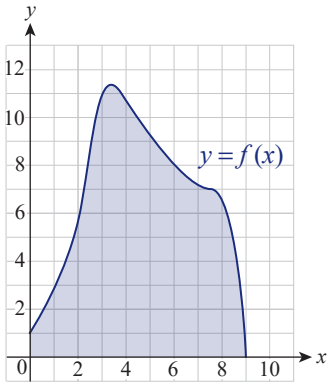
PRACTICE

The function $f(x)$ is given by its graph. Use the Trapezoidal Rule and Simpson's Rule, respectively, to approximate the shaded area $\int_a^b f(x)dx$ by **a.** T_6 and **b.** S_6.

1.



2.



Use the Trapezoidal Rule and Simpson's Rule with $n = 8$ to approximate the integral. Then find the exact value and compare your answers.

3. $\int_0^8 x^4\,dx$

4. $\int_1^5 \frac{1}{x}dx$

5. $\int_1^5 \frac{1}{x^2}dx$

6. $\int_0^4 \sqrt{x}\,dx$

7. $\int_0^4 x^3\,dx$

8. $\int_{-2}^6 \sqrt[3]{x+2}\,dx$

9. $\int_0^2 e^x\,dx$

10. $\int_1^5 \ln x\,dx$

11. $\int_{-2}^6 \left(4-\frac{1}{2}x\right)dx$

12. $\int_{-4}^4 \left(16-x^2\right)dx$

13. $\int_0^4 x\sqrt{x^2+2}\,dx$

14. $\int_0^{16} \frac{1}{\sqrt{x+1}}dx$

15. $\int_0^8 \frac{x}{\sqrt{x^2+1}}dx$

Use **a.** the Trapezoidal Rule and **b.** Simpson's Rule to approximate the definite integral for the indicated value of n.

16. $\int_0^4 \sqrt[4]{x}\,dx; \quad n=4$

17. $\int_0^5 \sqrt{x^4+4}\,dx; \quad n=10$

Some texts discuss the "Midpoint Rule" as a numerical integration method. The idea is simply forming a Riemann sum by choosing the midpoint of each subinterval as the sample point.

Use the "Midpoint Rule" with $n = 8$ to approximate the integral and compare your answers to those in Exercises 3–5.

18. $\int_0^8 x^4\,dx$

19. $\int_1^5 \frac{1}{x}dx$

20. $\int_1^5 \frac{1}{x^2}dx$

21. Use Simpson's Rule with $n = 6$ to approximate $\ln 2 = \int_1^2 \frac{1}{x}\,dx$.

22. Use Simpson's Rule with $n = 6$ to approximate $\pi = \int_0^1 \frac{4}{x^2+1}\,dx$.

23. Use Simpson's Rule with $n = 24$ to approximate the area of the region bounded by the graphs of $y = \sqrt{1+x^4}$, $x = -6$, $x = 6$, and the x-axis. (Notice that this problem leads to a nonelementary integral.)

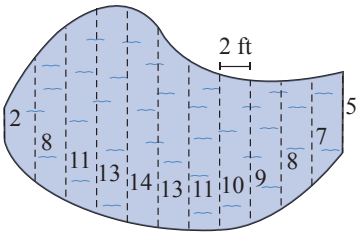
APPLICATIONS

24. The following table summarizes acceleration data for the 2012 Ford Mustang Boss 302 Laguna Seca. Use Simpson's Rule to estimate the total distance traveled by "the Boss" during its timed 0–120 mph run. (**Hint:** Sketching a graph similar to the one in Example 2 is useful. Be sure to identify which area you can approximate and how it yields the answer to the problem.)

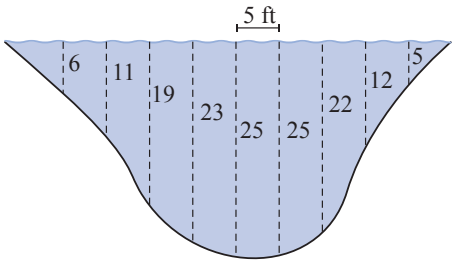
Time to Speed

Miles per Hour	Seconds
0–120	13.0
0–110	10.9
0–100	9.1
0–90	7.6
0–80	6.3
0–70	5.2
0–60	4.1
0–50	3.3
0–40	2.4
0–30	1.7
0–20	1.1
0–10	0.4

Source: *Road & Track*



25. Use the Trapezoidal Rule to estimate the amount of water needed to raise the water level by two inches in a pool with the shape shown in the figure. At 2-foot intervals, the distances across the pool (in feet) are as indicated in the diagram.



26. The figure shows the vertical cross-section of the Lazee river where the Dinkatown ferry docks. The depth of the river is indicated at 5-foot intervals in the diagram. If the river flows at 5 ft/s, use Simpson's Rule to estimate the amount of water passing by the dock every second.

WRITING & THINKING

27. Prove that Simpson's Rule actually gives the exact answer for definite integrals of all polynomials of degree 3 or less.