Thus the final form for $P(t)$ is $P(t)=\frac{10,000}{1+99e^{-0.14t}}$.

Table 1 contains the general solutions of differential equations that arise in three basic growth applications.

Application	Differential Equation	General Solution
Unbounded growth	$\frac{dy}{dt}=ky$	$y=Ce^{kt}$
Bounded growth	$\frac{dy}{dt}=k(M-y)$	$y=M+Ce^{-kt}$
Logistic curve	$\frac{dy}{dt}=ky(M-y)$	$y=\frac{M}{1+Ce^{-Mkt}}$

TABLE 1: General Solutions for Growth Applications

14.7 EXERCISES

PRACTICE

In Exercises 1–12, verify that the differential equation has the given function as a particular solution.

1. $\frac{dy}{dx}=6,\ y=6x-1$

2. $\frac{dy}{dx}=3x-2,\ y=\frac{3}{2}x^2-2x-4$

3. $\frac{dy}{dx}=3+y,\ y=e^x-3$

4. $x\frac{dy}{dx}-x+2=0,\ y=x-2\ln x+7$

5. $\frac{dy}{dx}=y^{\frac{3}{2}},\ y=\frac{4}{(5-x)^2}$

6. $\frac{dy}{dx}=-0.5y,\ y=3e^{-0.5x}$

7. $2\frac{dy}{dx}+3y=1,\ y=\frac{1}{3}-2e^{-1.5x}$

8. $x\frac{dy}{dx}=xy+y,\ y=4xe^x$

9. $2x^2y''-xy'-2y=4-15x,\ y=3x^2+5x-2$

10. $x^2y''+xy'-y+\ln x=0,\ y=x+\ln x$

11. $x^2y''-xy'+y=0,\ y=x\ln x$

12. $y''-2y'+y=0,\ y=e^x(x+2)$

In Exercises 13–20, find the solution of each separable differential equation.

13. $\frac{dy}{dx}=3x+\frac{1}{x}$

14. $\frac{dy}{dx}=-3xy$

15. $\frac{dy}{dx}=\frac{2y-1}{x+1}$

16. $x\frac{dy}{dx} = \frac{x^2+1}{y^2}$

17. $\frac{dy}{dx} = -0.4y$

18. $\frac{dy}{dx} = -2(26-y)$

19. $\frac{dy}{dx} = (1-5x)y^2$

20. $\frac{dy}{dx} = (x^2+1)e^{-y}$

In Exercises 21–32, solve each initial-value problem or obtain a general solution as indicated. (Refer to Table 1 in the text if necessary.)

21. $\frac{dy}{dx} = 0.6y(20-y)$

22. $\frac{dy}{dx} = 0.3y(50-y)$

23. $\frac{dy}{dx} = 0.25y,\ y = 5$ when $x = 0$

24. $\frac{dy}{dx} = -3x,\ y = 10$ when $x = 2$

25. $x\frac{dy}{dx} = y+1,\ y = 14$ when $x = 3$

26. $\frac{dy}{dx} = -2xy,\ y = 18$ when $x = 0$

27. $\frac{dy}{dx} = -6x^2y^2,\ y = 2$ when $x = 1$

28. $\frac{dy}{dx} = \frac{xy}{x^2+1},\ y = 7$ when $x = 0$

29. $\frac{dy}{dx} = 8x+2xy,\ y = 10$ when $x = 0$

30. $\frac{dy}{dx} = 0.3(80-y),\ y = 60$ when $x = 0$

31. $\frac{dy}{dx} = 0.8y(40-y),\ y = 30$ when $x = 0$

32. $\frac{dy}{dx} = 0.04y(60-y),\ y = 10$ when $x = 0$

APPLICATIONS

33. Elasticity of demand: The elasticity of demand for a product is given by $E = 1.5$. Find the demand function $p = D(x)$ if $D(8) = 24$.

34. Elasticity of demand: The elasticity of demand for a product is given by $E = 2$. Find the demand function $p = D(x)$ if $D(25) = 30$.

35. Elasticity of demand: The elasticity of demand for a product is given by $E = \frac{2(120-x)}{x}$. Find the demand function $p = D(x)$ if $D(20) = 180$.

36. Elasticity of demand: The elasticity of demand for a product is given by $E = \frac{60-0.4x}{0.2x}$. Find the demand function $p = D(x)$ if $D(70) = 28$.

37. **Resale value:** The resale or salvage value V of a machine decreases at a rate proportional to its value. Thus $\frac{dV}{dt} = -kV,$ where t is the machine's age in years and k is its rate of decrease in value.
   a. Find the expression for the value when the machine is t years old if the original value was \$24,000 and the rate of decrease is 6 percent.
   b. Find the value of the machine when it is 7 years old.

38. **Drug concentration:** The amount A of a drug remaining in a body t hours after an injection decreases at a rate proportional to the amount present. This suggests the differential equation $\frac{dA}{dt} = -kA.$ The amount of a certain drug decreases at a rate of 3 percent per hour. Find the amount of the drug remaining in the body 4 hours after an injection of 20 cc of the drug.

39. **Newton's Law of Cooling:** Newton's Law of Cooling states that the rate at which the temperature T of an object changes is proportional to the difference between the temperature of the object and the temperature of the surrounding medium. That is $\frac{dT}{dt} = -k(T - M),$ where k is the constant of proportionality, t is time, and M is the constant temperature of the medium.
   a. Solve the differential equation for $T(t)$.
   b. Find $T(5)$, if $T(0) = 78°$, $M = 26°$, and $k = 0.3$.

40. **Newton's Law of Cooling:** The temperature of a roast was 160° when it was removed from an oven and placed in a room with constant temperature of 76°. After 10 minutes, the temperature of the roast was 152°. Find the temperature 20 minutes after the roast was removed from the oven. (See Exercise 39.)

41. **Spread of a rumor:** In a small community with a population of 2800, a rumor about the mayor was started. The rate at which the rumor spread was approximated by $\frac{dN}{dt} = 0.0003N(2800 - N)$ people per day, where N is the number of people who have heard the rumor t days after the rumor was started.
   a. Write an equation for $N(t)$, assuming that 20 people have heard the rumor at $t = 0$.
   b. How many days will it take for 1500 people to hear the rumor? Round to the nearest day.

42. **Spread of a disease:** The population of seals on an island is about 600. Biologists estimate that there are 12 seals with a very infectious disease. The disease will spread at a rate $\frac{dN}{dt} = 0.006N(600 - N)$ seals per day, where N is the number of infected seals t days after the discovery of the disease.
   a. Write a function $N(t)$ for the number of seals infected t days after the discovery of the disease.
   b. At what time t will 300 seals will be infected? Round to the nearest day.