

14.2 EXERCISES

 PRACTICE

In Exercises 1–36, use the technique of substitution to perform each integration.

1. $\int (x+4)^7 dx$

2. $\int (y-6)^{-3} dy$

3. $\int 2x(x^2-1)^{\frac{1}{2}} dx$

4. $\int 3x^2(x^3-5)^{\frac{1}{3}} dx$

5. $\int \frac{1}{t+2} dt$

6. $\int \frac{1}{x-11} dx$

7. $\int \frac{3t^2}{t^3+4} dt$

8. $\int \frac{1}{y^2-8} \cdot 2y dy$

9. $\int e^{y+5} dy$

10. $\int e^{x-9} dx$

11. $\int e^{-0.2x} (-0.2) dx$

12. $\int e^{0.5t} (0.5) dt$

13. $\int \frac{1}{5x+3} dx$

14. $\int (3y-2)^{-2} dy$

15. $\int \frac{1}{\sqrt{4x-1}} dx$

16. $\int e^{-4x} dx$

17. $\int xe^{2x^2} dx$

18. $\int \frac{x}{2x^2+5} dx$

19. $\int \frac{x}{(3x^2-1)^2} dx$

20. $\int y^2 e^{-2y^3} dy$

21. $\int \frac{2t+1}{t^2+t-4} dt$

22. $\int (x^2+3x-1)^4 (2x+3) dx$

23. $\int 5ye^{-y^2} dy$

24. $\int \frac{e^{\sqrt{t}}}{\sqrt{t}} dt$

25. $\int 6x\sqrt{5+2x^2} dx$

26. $\int \frac{e^t}{e^t-1} dt$

27. $\int \frac{4}{x^2} e^{\frac{1}{x}} dx$

28. $\int 4y^3 \sqrt{3y^2+7} dy$

29. $\int e^{2x} (1-3e^{2x})^2 dx$

30. $\int \frac{4x+10}{x^2+5x+2} dx$

31. $\int \frac{\ln x}{x} dx$

32. $\int \frac{\ln 4x}{x} dx$

33. $\int \frac{1}{x \ln x} dx$

34. $\int \frac{(\ln x)^2}{x} dx$

35. $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$

36. $\int \frac{7x}{e^{x^2}} dx$

In Exercises 37–40, divide first and then integrate.

37. a. $\int \frac{x-1}{x-2} dx$ **(Hint: $\frac{x-1}{x-2} = 1 + \frac{1}{x-2}$)**

b. $\int \frac{x+3}{x+1} dx$ **(Hint: $\frac{x+3}{x+1} = 1 + \frac{2}{x+1}$)**

38. a. Rework Exercise 37a and substitute $u = x - 2$.b. Rework Exercise 37b and substitute $u = x + 1$.

39. $\int \frac{x+2}{x+4} dx$

40. $\int \frac{x+5}{x-3} dx$

In Exercises 41–44, find a function $f(x)$ given $f'(x)$ and one (x, y) -value.

41. $f'(x) = 10xe^{5x^2}$; $(0, 11)$

42. $f'(x) = \frac{3}{x-5}$; $(6, 5)$

43. $f'(x) = (2x+2)^5$; $(-1, 10)$

44. $f'(x) = \frac{12x^2}{(x^3+6)^2}$; $(0, -\frac{2}{3})$

APPLICATIONS

45. **Appreciation:** The value V of a painting is increasing at a rate of $4500(25 - 1.8t)^{-\frac{3}{2}}$ dollars per year. The painting originally sold for \$1000.

- Write a function for its value t years after the original sale.
- What will the painting be worth 5 years after the original sale?

46. **Skills development:** It is estimated that after t weeks of practice, students in a typing class can increase their speed $24e^{-0.2t}$ words per minute for each additional week of practice.

- If their initial speed was 0 words per minute ($S(0) = 0$), write a function to represent their speed after t weeks of practice.
- How fast can they type after 10 weeks (to the nearest word)?

47. **Price:** After the NBA championship basketball game, the price of a souvenir cap changes at the rate of $-\frac{1}{2x}$ dollars per cap, where x is the number of caps sold (in thousands). Write a function for the price p if $p(10) = 12.85$.

48. **Profit:** The marginal profit from the production and sale of x units of a product is estimated to be $\frac{100}{1+0.5x} - 4.5$ dollars per unit. If $P(0) = -60$, find the profit function $P(x)$.

- 49. Daily production:** Records show that t hours after starting work on a typical day, an employee can assemble bikes at a rate of $\frac{6t+15}{\sqrt{t^2+5t}}$ per hour. Find the daily production function $N(t)$ if $N(0) = 0$.
- 50. Position of a particle:** A particle is moving in a straight line with the velocity $v(t) = \sqrt{2t+7}$ feet per second, where t represents time in seconds. Find the position function $s(t)$ if $s(0) = 0$.
- 51. Position of a projectile:** The velocity of a projectile moving in a straight line t seconds after it is fired is given by $v(t) = 36 + \frac{60}{(t+1)^2}$ feet per second. Find the position function $s(t)$ if $s(1) = 10$.
- 52.** The marginal value for a tract of land is $V' = \frac{20}{2t+1}$, where t is time in years since 2000 and V is in thousands of dollars.
 a. Determine the function V in terms of t if $V(0) = 20$.
 b. What was the value of the land in 2015?
- 53.** A point mass moving on a horizontal axis has a deceleration given by $a(t) = -48(2t+1)^2$, where t is time in seconds and a is in feet per second per second.
 a. If $v(0) = 6$ ft/s, determine a velocity function $v(t)$.
 b. If $s(0) = -\frac{3}{4}$ feet, determine a distance function $s(t)$.
- 54.** The number of viewers of a new TV show grew at a rate $V'(t) = 900e^{0.3t-4}$ all summer long, where V is the total number of viewers in thousands and t is the number of weeks since June 1st.
 a. Determine $V(t)$ if $V(0) = 100$.
 b. At what rate is V changing when $t = 5$?
 c. When will the show hit 1,000,000 viewers ($V = 1000$)?
- 55.** Suppose in a medieval country, from 1200 to 1300, the life expectancy of a female serf changed at the rate $f'(t) = \frac{0.3}{1+0.01t}$, where t is time in years after 1200 and $f(t)$ is the average age of death.
 a. What are the units of $f'(t)$?
 b. Determine $f(t)$ if $f(0) = 30$.
 c. What was the life expectancy of a female serf in 1300?
- 56.** A new evening school is growing at the rate of $p'(t) = \frac{150}{\sqrt{1+0.2t}}$, where $p(t)$ is the total evening school student population and t is the time in years after 2000. The initial enrollment was 1500 students.
 a. How fast was enrollment changing in 2002?
 b. What was the expected enrollment in 2005?