13.6 EXERCISES

PRACTICE

Find the differential for each of the functions in Exercises 1–14.

1. $y = x^3 + 5$

2. $y = 4x^3 + x - 7$

3. $u = (2t^2 + 1)^2$

4. $u = (5t + 9)^2$

5. $A = \pi r^2$

6. $A = x(44 - 2x)$

7. $V = \frac{4}{3}\pi r^3$

8. $V = x^3$

9. $S = 4x^2 + \frac{1350}{x}$

10. $C = 75 + 10x - 0.6\sqrt{2x}$

11. $C = 40 + 3x + 0.4\sqrt{x}$

12. $S = 2\pi r^2 + \frac{90\pi}{r}$

13. $P = -0.2x^2 + 75x - 2400$

14. $P = -0.3x^2 + 84x - 870$

For each of the functions given in Exercises 15–22, use the given values for x and Δx to find **a.** Δy, **b.** dy, and **c.** $\Delta y - dy$.

15. $y = x^2 - 3x + 4,\ x = 3,\ \Delta x = 0.2$

16. $y = x^2 + 5x - 9,\ x = 2,\ \Delta x = 0.15$

17. $y = (2x^2 - 4)^3,\ x = -2,\ \Delta x = 0.04$

18. $y = (x^2 + x - 1)^3,\ x = -3,\ \Delta x = 0.05$

19. $y = 20\left(x - \frac{24}{x}\right),\ x = 2,\ \Delta x = -0.12$

20. $y = 2x^2 + \frac{125}{x^2},\ x = 5,\ \Delta x = -0.15$

21. $y = \sqrt{3x + 4},\ x = 7,\ \Delta x = 0.5$

22. $y = \sqrt{12 - 5x},\ x = 2,\ \Delta x = 0.07$

Use differentials to approximate the indicated roots in Exercise 23–30. Express your answer as $a \pm b/c$, where a is the nearest integer.

23. $\sqrt{37}$

24. $\sqrt{65}$

25. $\sqrt[3]{26}$

26. $\sqrt[3]{126}$

27. $\sqrt{50.4}$

28. $\sqrt{79.5}$

29. $\sqrt[3]{62.3}$

30. $\sqrt[3]{218.3}$

APPLICATIONS

31. Cost: A total cost function (in dollars) is given by $C(x) = 375 + 9x + 0.01x^2$. Use differentials to estimate the change in cost when the level of production is increased from 60 to 62 units.

32. Cost: A total cost function (in dollars) is given by $C(x) = 930 + 15x + 0.2x^2$. Using differentials, estimate the change in cost from $x = 100$ to $x = 101$.

33. Profit: The weekly revenue (in dollars) from the sale of x coffee makers is given by $R(x) = 40x$. The total cost function is given by $C(x) = 370 + 16x + 0.2x^2$. Use differentials to approximate the change in profit if the weekly sales are increased from 25 to 28 coffee makers.

34. Profit: The monthly revenue from the sale of x 50-gallon aquariums is given by $R(x) = 54x - 0.3x^2$ dollars. The total cost function is given by $C(x) = 0.1x^2 + 4x + 200$ dollars. Using differentials, find the approximate change in profit if the monthly sales are increased from 40 to 44 aquariums.

35. Population: It is estimated that t years from now the population of a city will be $P(t) = 10(4000 + 2t^2) - 1600t$. Use differentials to estimate the change in population as t changes from 6 to 6.25 years.

36. Bacterial population: It is estimated that t hours from now the population of bacteria in a culture will be $P(t) = \frac{8000}{\sqrt{8 - 0.5t}}$. Use differentials to estimate the change in population as t changes from 8 to 8.3 hours.

37. Volume: The edge of a cube measures 18 in. with a possible error in measurement of 0.02 in. Use differentials to estimate the possible error in computing the volume.

38. Fiberglass coating: A cube is 12 in. on a side. It is to be covered with a fiberglass coating 0.25 in. thick. Use differentials to estimate the volume of the fiberglass coating.

39. Measurement error: A manufacturer of cargo containers receives an order for a cube-shaped container. The specifications state that the volume should be 125 ft^3. with a maximum error of no more than 1 ft^3. Using differentials, find the possible error in the length of the edges.

40. Melting ice: A block of ice is in the form of a 10 in. cube. If it melts uniformly until the volume changes to 972 in.^3, approximate the change in the length of each edge by using differentials.

41. Volume of a weather balloon: A spherical weather balloon is being inflated. Use differentials to find the approximate change in the volume if the radius changes from 20 to 21.5 inches. (**Hint:** $V = \frac{4}{3}\pi r^3$.)

42. Volume of a tumor: A spherical cancer tumor is being treated with an experimental drug. The radius of the tumor has been reduced from 1.6 to 1.4 cm. Use differentials to estimate the change in the volume of the tumor. (**Hint:** $V = \frac{4}{3}\pi r^3$.)