Solution

a. The quantity demanded is x and $x = F(p)$.

$$\begin{aligned} x &= F(1.70) \\ &= 180 - 30(1.70) \\ &= 129 \end{aligned}$$

b. $E = -\dfrac{pF'(p)}{F(p)} = -\dfrac{p(-30)}{180 - 30p} = \dfrac{p}{6 - p}$

c. $E(1.70) = \dfrac{30(1.70)}{180 - 30(1.70)} = \dfrac{51}{129} = \dfrac{17}{43}$

d. Since $E < 1$, the demand is inelastic and so an increase in price will bring a decrease in demand and an increase in revenue.

e. $$\begin{aligned} R = xp &= (180 - 30p)p \\ &= 180p - 30p^2 \end{aligned}$$

Thus $R' = 180 - 60p$. Setting $R' = 0$, we obtain $60p = 180$, so $p = \$3.00$ per loaf.

Since $R'' = -60$, the function R is concave down at $p = 3$ and this price gives a maximum for the revenue.

13.4 EXERCISES

PRACTICE

For each of the demand functions in Exercises 1–16, find **a.** the function describing the elasticity of demand and **b.** the value of x that maximizes the revenue.

1. $p = D(x) = 84 - 3x$

2. $p = D(x) = 144 - 1.5x$

3. $p = D(x) = 520 - 2.6x$

4. $p = D(x) = 480 - 3.2x$

5. $p = D(x) = 200e^{-0.2x}$

6. $p = D(x) = 67e^{-0.1x}$

7. $p = D(x) = 88e^{-0.025x}$

8. $p = D(x) = 130e^{-0.04x}$

9. $p = D(x) = \sqrt{150 - x}$

10. $p = D(x) = \sqrt{180 - 2x}$

11. $p = D(x) = \sqrt{162 - 3x}$

12. $p = D(x) = \sqrt{255 - 2.5x}$

13. $p = D(x) = 18 - \sqrt{x}$

14. $p = D(x) = 21 - 2\sqrt{x}$

15. $p = D(x) = 363 - x^2, \quad x \le 18$

16. $p = D(x) = 600 - 0.5x^2, \quad x \le 34$

APPLICATIONS

17. Maximum revenue: The demand function for an electric pencil sharpener is given by $p = D(x) = 19.2 - 0.4x$ dollars. Find the level of production for which the revenue is maximized.

18. Maximum revenue: The demand function for a popular stereo receiver is given by $p = D(x) = 540 - 0.05x^2$ dollars. Find the level of production for which the revenue is maximized.

19. Elastic demand: The demand function for an exclusive wool blanket is given by $p = D(x) = 33 - 2\sqrt{x}$ dollars, where x is in thousands of blankets. Find the level of production for which the demand is elastic.

20. Elastic demand: Find the levels of production for which the demand is elastic if the demand is given by $p = D(x) = \sqrt{207 - 3x}$ dollars.

21. Elastic demand: An arcade sells video games and determines that $x = 30\left(1 - e^{-\frac{p}{10}}\right)$, where x is the number of video games demanded for a unit price p.

a. Determine the quantity demanded when $p = \$10$ per game.
b. Determine E and interpret the result at $p = \$10$.
c. What revenue is generated at $p = \$10$?

22. Elastic demand: Lucky Blooms sells a new rose variety which has established a demand of $x = f(p) = \dfrac{e^{\frac{p}{3}} + 350}{e^{\frac{p}{2}}}$.

a. Determine the quantity demanded when $p = \$3$.
b. Determine E and interpret the result at $p = \$3$.

23. Elastic demand: The demand for a product is given by $x = F(p) = \dfrac{1800}{10 + \ln(1 + p)}$.

a. If $p = 20$, determine E and interpret the results.
b. What is the revenue function R?
c. Use R' to determine if R is increasing at $p = 20$. Is your answer consistent with part **a.**?

24. Elastic demand: Suppose a product has a demand function $x = F(p) = 300e^{-\frac{p}{10}}$.

a. Find the elasticity function.
b. Is the demand elastic or inelastic at $p = \$20$?
c. Determine the unit price which maximizes revenue.
d. Discuss whether or not your answers to **b.** and **c.** are consistent.

25. Elastic demand: Suppose the demand for a product is $p = D(x) = 300e^{-\frac{x^2}{200}}$.

a. Determine the unit price if the quantity $x = 5$.
b. What is formula for $D'(x)$?
c. What is the elasticity at $x = 5$?
d. Determine the value of x which maximizes revenue.