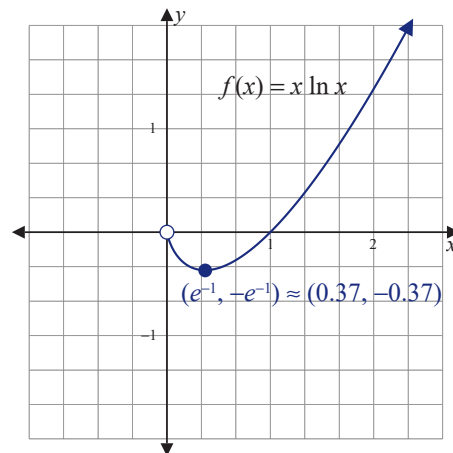


A local (and absolute) minimum occurs at the critical value $x = e^{-1}$, and

$$\begin{aligned} f(e^{-1}) &= e^{-1} \ln e^{-1} \\ &= e^{-1}(-1) = -e^{-1}. \end{aligned}$$

Note: One easy point to get without a calculator is $(1, 0)$ (recall that $\ln 1 = 0$). Therefore, it is possible to notice that $1 \cdot \ln 1 = 0$, and $(1, 0)$ is an x -intercept.



13.1 EXERCISES

💡 PRACTICE

Find the derivative of each of the functions in Exercises 1–28.

1. $f(x) = x^2 - \ln x$

2. $f(x) = 4x^2 + \ln x^2$

3. $f(x) = 25 + x \ln x$

4. $f(x) = (x^2 + 1) \ln x$

5. $y = \frac{\ln x}{x}$

6. $y = \frac{x^2}{\ln x}$

7. $y = (\ln x)^3$

8. $y = \sqrt{\ln x}$

9. $f(x) = \ln(5x + 3)$

10. $f(x) = \ln(7x - 2)$

11. $f(x) = \ln(x^2 + 2)$

12. $f(x) = \ln(x^2 + 3x)$

13. $f(x) = \ln \sqrt{2x^2 - 1}$

14. $f(x) = \ln \sqrt{x^3 + 4}$

15. $y = \ln(4x + 3)^2$

16. $f(x) = \ln(x^2 - 4)^3$

17. $y = \sqrt{x} \ln(x^2 + 2)$

18. $y = \frac{\ln(5x + 2)}{x^3}$

19. $y = \frac{\ln(x^2 + 2x - 1)}{x}$

20. $y = x^{-2} \ln(x^2 - 3x + 4)$

21. $f(x) = \ln((3x + 1)(x^2 + 3))$

22. $f(x) = \ln(x^2(4x - 1))$

23. $f(x) = \ln((2x - 1)^2(x^2 - 2))$

24. $f(x) = \ln(\sqrt{4x - 7}(6x + 7))$

25. $y = \ln\left(\frac{x+1}{x-2}\right)$

26. $y = \ln\left(\frac{3x-1}{x+5}\right)$

27. $f(x) = \ln\left(\frac{x^2 - 5}{2x + 9}\right)$

28. $f(x) = \ln \sqrt{\frac{4x+3}{x^2-6}}$

For Exercises 29–34, determine a formula for $f''(x)$. Use your calculator (if necessary) to solve the equation $f''(x) = 0$ and locate any possible inflection points.

29. $f(x) = \frac{x}{\ln x}$

30. $f(x) = (2x + x^2) \ln x$

31. $f(x) = 3x^2 \ln x$

32. $f(x) = \ln(x^3)$

33. $f(x) = (\ln x)^3$

34. $f(x) = \ln x + x^2 + 3x + 2$

Use logarithmic differentiation to find the derivatives of the functions in Exercises 35–42.

35. $y = (2x - 5)^3 \sqrt{x^2 - 2x}$

36. $y = (4 - 5x)^4 (7x + 2)^{-\frac{2}{3}}$

37. $y = (x^2 + 2)^4 (x^2 - 1)^{-\frac{1}{3}}$

38. $y = (3x - 2)^5 \sqrt{x^2 + x}$

39. $y = \frac{(2x + x^2)^3}{(4x - 9)^2}$

40. $y = \frac{\sqrt[3]{x^2 + 4}}{(2 - 5x)^3}$

41. $y = \frac{(x + 2)^2 (3x + 4)^2}{\sqrt{x - 6}}$

42. $y = \frac{(x^2 - 3)^2 \sqrt{x^2 + 3x}}{x + 7}$

In Exercises 43–48, find the absolute extrema for each of the functions on the indicated interval.

43. $f(x) = x - \ln x; [0.5, 2]$

44. $f(x) = x^2 - 8 \ln x; [0.3, 4]$

45. $f(x) = \frac{x}{\ln x}; [1.2, 3]$

46. $f(x) = \frac{\ln x}{x^2}; [1, 2]$

47. $f(x) = x^2 - \ln x^3; [1, e]$

48. $f(x) = \ln(3 + 2x - x^2); [0, 2.5]$

Using the graph-sketching techniques discussed in Chapter 12, sketch the graph of each function in Exercises 49–54.

49. $f(x) = 4x \ln x$

50. $f(x) = x^2 \ln x$

51. $f(x) = x - 3 \ln x$

52. $f(x) = 4x - \ln x^2$

53. $f(x) = \frac{\ln x}{x}$

54. $f(x) = \frac{\ln x}{x^2}$

 APPLICATIONS

- 55. Marginal revenue:** A retailer has determined that the revenue from the sale of x end tables is given by the function $R(x) = 96x + \ln(4x^2 + 15)$ dollars. Find the marginal revenue.
- 56. Advertising:** An automobile dealer has estimated that he can sell $N(x) = 420 + 72\ln(1 + 0.5x)$ cars annually, where x (in thousands of dollars) is the amount spent on advertising.
- Find the number of cars sold if \$6000 is spent on advertising.
 - Find the rate of change in number of cars sold if \$6000 is spent on advertising.
- 57. Revenue:** The daily demand for a product is given by $p = -8\ln(0.01x)$, where p is the price in dollars when x units are sold and $0 < x \leq 100$. Find the maximum daily revenue.
- 58. Revenue:** The demand for a product is given by $p = 14 - 6.5\ln x$, where x is the number of units (in thousands) that can be sold at a price p dollars and $2 \leq x \leq 8$. Find the maximum revenue.
- 59. Insect population:** Mediterranean fruit flies are discovered in a citrus orchard. The Department of Agriculture has determined that the population of flies t hours after the orchard has been sprayed with pesticide is approximated by $N(t) = 25 - 5t\ln(0.04t) - t$, where $0 < t \leq 25$.
- Find $N(3)$, $N(10)$, $N(25)$.
 - What will be the maximum number of flies in the orchard?
- 60. Air quality:** The Air Quality Management Board estimates that t hours after midnight in a major city the level of ozone in the air is about $N(t) = 0.013 - 0.007t\ln(0.026t)$ parts per million, where $0 < t \leq 18$.
- Find $N(6)$, $N(10)$, $N(18)$.
 - Find the maximum level of pollution.