b. The projectile hits the ground when $s(t) = 0$.

$$-16t^2 + 160t + 176 = 0 \qquad \text{Solve } s(t) = 0 \text{ for } t.$$
$$-16\left(t^2 - 10t - 11\right) = 0$$
$$-16(t - 11)(t + 1) = 0$$
$$t = 11 \text{ or } \cancel{t = -1} \qquad \text{Time cannot be negative, so we discard the } t = -1.$$

Thus the projectile hits the ground 11 seconds after it is fired.

c. The projectile's velocity when it hits the ground is $v(11)$. From part **a.** we know

$$v(t) = -32t + 160.$$

So

$$v(11) = -32(11) + 160$$
$$= -352 + 160$$
$$= -192 \text{ ft/sec}. \qquad \text{The negative sign indicates the projectile is falling.}$$

The projectile will be falling at a rate of 192 ft/sec when it hits the ground.

12.6 EXERCISES

APPLICATIONS

1. **Volume of a box:** A rectangular box with no top is to be made from a piece of cardboard that is 24 in. by 24 in. Equal squares are to be cut from each corner and the sides folded to form the box. What size should the squares be to maximize the volume of the box?

2. **Volume of a box:** A rectangular box with no top is to be made from a piece of cardboard that is 20 in. by 20 in. Equal squares are to be cut from each corner and the sides folded to form the box. What size should the squares be to maximize the volume of the box?

3. **Volume of a box:** Equal squares are to be cut from each corner of a rectangular piece of thin sheet metal, and the sides are to be folded to form a box. If the piece of metal is 8 in. by 15 in., find the dimensions of the box having maximum volume.

4. **Volume of a box:** Equal squares are to be cut from each corner of a rectangular piece of thin sheet metal, and the sides are to be folded to form a box. If the piece of metal is 10 in. by 16 in., find the dimensions of the box having maximum volume.

5. **Surface area:** A rectangular box is designed to have a square base and an open top. The volume is to be 864 in.^{3}
   a. What dimensions will give a minimum surface area?
   b. What is the minimum surface area?

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6. **Surface area:** A rectangular box is designed to have a square base and an open top. The volume is to be 256 in.^{3}
   a. What dimensions will give a minimum surface area?
   b. What is the minimum surface area?

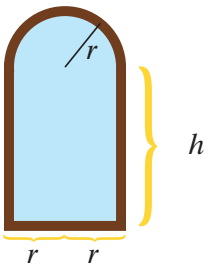
7. **Container design:** A container manufacturer is asked to design a closed rectangular shipping crate with a square base. The volume is 10 ft^3. The material for the top and sides costs \$2 per square foot and the material for the bottom costs \$3 per square foot. Find the dimensions of the box that will minimize the total cost.

8. **Container design:** A container manufacturer is asked to design a closed rectangular shipping crate with a square base. The volume is 36 ft^3. The material for the top costs \$1 per square foot, the material for the sides costs \$0.90 per square foot, and the material for the bottom costs \$1.40 per square foot. Find the dimensions of the box that will minimize the total cost of material.

9. **Container design:** An investor plans to manufacture rectangular box containers whose bottom and top measure x by $3x$. The box must contain 18 cubic feet. The top and bottom will cost \$2 per square foot, and the four sides will cost \$3 per square foot. What should the height h be so as to minimize costs?

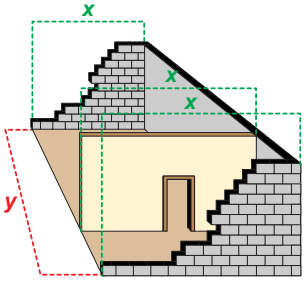
10. **Area:** A rectangular plot is to be enclosed with an existing block wall as one side. If there are 680 ft of fencing available for the other three sides, find the dimensions that will maximize the area.

11. **Area:** A rectangular play area is to be enclosed with the side of a house as one of the sides. If there are 74 ft of fencing available for the other three sides, find the dimensions that will maximize the area.

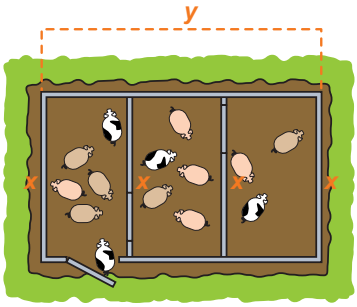


12. **Window area:** A front window on a new home is designed as a rectangle with a semicircle on the top. If the window is designed to let in a maximum amount of light, and the architect fixes the perimeter of the entire window at 600 inches, determine the radius r and rectangular height h so as to maximize the area.

13. **Wall construction:** An old stone wall makes two legs of a right angle, one 40 feet long and the other 20 feet long. A constructor is told to add 220 feet of new stone fence to complete a rectangular fence. How should he complete the fence so as to maximize the enclosed area? Determine the maximum enclosed area he may obtain.



14. **Construction:** A warehouse is being constructed with a total floor area of 2200 ft^2. A single partition is built to divide the building into storage area and office space. The exterior walls cost \$160 per foot, and the interior wall costs \$120 per foot. Find the dimensions of the warehouse that will minimize the cost.



15. **Construction:** A farmer wants to build a rectangular pen and then divide it with two interior fences. The total area is to be 2484 ft^2. The exterior fence costs \$18 per foot, and the interior fence costs \$16.50 per foot. Find the dimensions of the pen that will minimize the cost.

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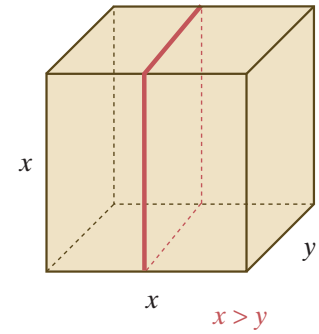
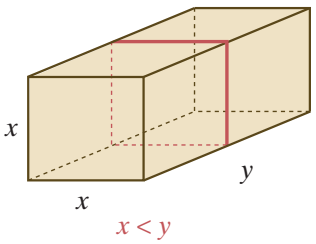
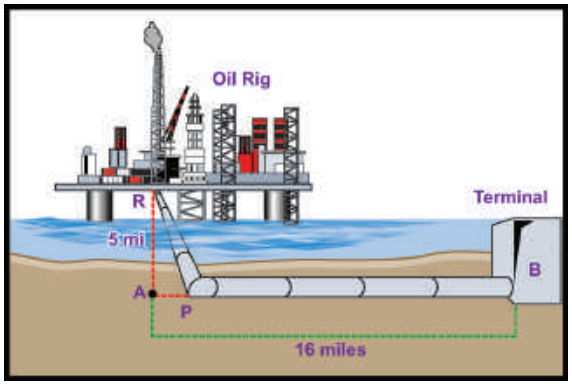




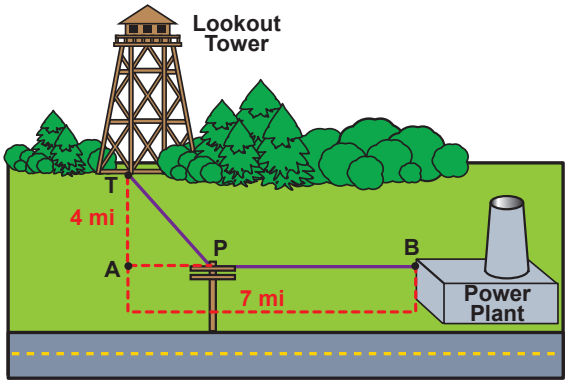
16. Shipping: The Postal Service has a limit of 108 in. on the combined length and girth of a rectangular package to be sent by parcel post. Find the dimensions of the package of maximum volume that has a square cross section. (**Hint:** There are two different answers, depending on the shape of the box. The two shapes are shown here. The girth is defined to be the smallest perimeter of a rectangular cross section of the box.)

17. Shipping: An independent parcel service has a limit of 130 in. on the combined length and girth of a rectangular package it will ship. Find the dimensions of the package of maximum volume that has a square cross section. (**Hint:** There are two different answers. The girth is defined to be the smallest perimeter of a rectangular cross section of the box.)

18. Pipeline construction: An oil company wishes to run a pipeline from a drilling platform located 5 miles offshore to a shipping terminal 16 miles down the coast. The costs are $130,000 per mile to lay the pipeline underwater and $120,000 per mile to lay the pipeline over land. Find the location of point P (as illustrated in the diagram) so that the total cost of laying pipe will be minimized.



19. Power line construction: The U.S. Forest Service wishes to run a power line to a fire lookout tower located in a wooded area. The tower is 4 miles from the nearest road and the power source is 7 miles down that road. It costs $5000 per mile to run the line through the forest and $3000 per mile to run the line along the road. Find the location of point P (as illustrated in the diagram) so that the total cost of running the power line will be minimized.



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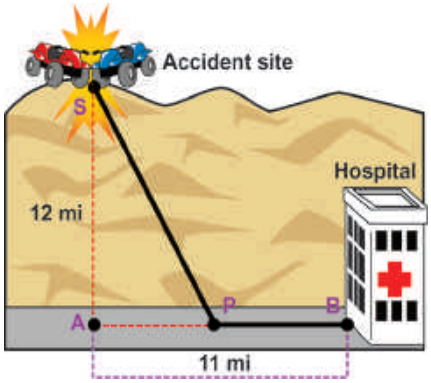
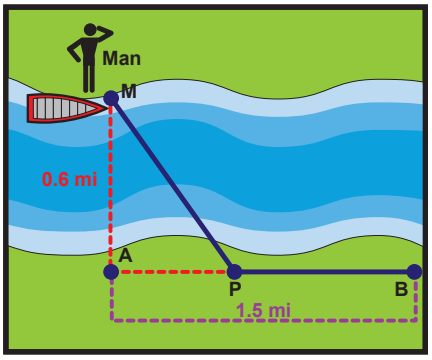
20. Minimum time: The Off-Roaders, an all-terrain vehicle club, were driving their four-wheelers in the desert when one member had a serious accident. At the time, they were 12 miles from the nearest paved road. The nearest hospital was located 11 miles down the paved road. If they can average 20 mph on the desert and 52 mph on the road, locate point P on the road toward which they should drive in order to minimize the time needed to get to the hospital.



21. Minimum time: A man is on the bank of a river that is 0.6 miles wide. He wants to reach a point on the opposite shore that is 1.5 miles downstream. If he can row a boat across the river at 4 mph and walk at 5 mph, find the location P, on the opposite shore, to which he should row in order to minimize the total time he would need to reach the point downstream.



22. Distance and velocity: A particle is moving along a straight line such that the distance traveled at the end of t seconds is given by $s(t) = 7t^2 + 30t$ feet.

a. Find the velocity if $t = 2$ seconds.

b. How far has the particle traveled?

23. Distance and velocity: A ball is rolled down an incline. The distance (in feet) of the ball from the starting point after t seconds is given by $s(t) = 19t + 8t^2$.

a. Find the velocity after 3 seconds.

b. How far has the ball traveled in 3 seconds?

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24. Distance and velocity: A projectile is fired vertically, and its height (in feet) after t seconds is given by $s(t) = 104t - 16t^2$.

a. Find the maximum height of the projectile.
b. When does the projectile hit the ground?
c. How fast is the projectile moving when it hits the ground?

25. Distance and velocity: A stone is projected vertically. The height (in feet) of the stone at time t (in seconds) is given by $s(t) = -16t^2 + 112t + 128$.

a. What is the maximum height of the stone?
b. When will the stone hit the ground?
c. What is the speed of the stone when it hits the ground?

26. Distance and velocity: A child rolls a hoop down a hilly street. The distance traveled (in feet) after t seconds is given by $s(t) = 4t + t^2$.

a. How far has the hoop traveled in 3 seconds?
b. What was the speed at 3 seconds?
c. At what rate was the speed changing at $t = 3$?

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