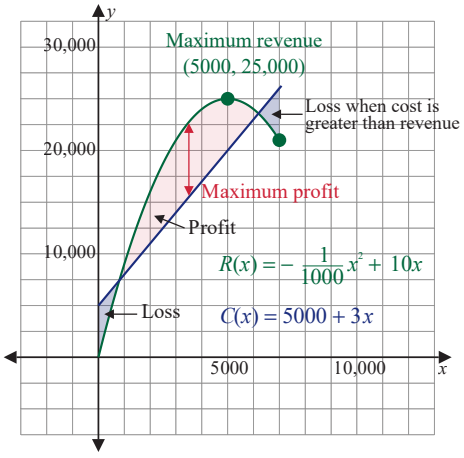
$$P'(x) = -\frac{1}{500}x + 7$$



Set $P'(x)$ equal to 0, and solve for x.

$$-\frac{1}{500}x + 7 = 0$$
$$-x = -3500$$
$$x = 3500$$

Thus, a maximum profit occurs if the company produces and sells 3500 calculators, which is only half its production capabilities. The price for each calculator would be

$$p = -\frac{1}{1000}(3500) + 10 = \$6.50.$$

The graph shown illustrates the relationship between profit, revenue, and cost. Note that maximum revenue and maximum profit do not necessarily occur at the same level of production and sales.

12.5 EXERCISES

APPLICATIONS

1. **Minimizing inventory costs:** An appliance store owner estimates that he will sell 125 vacuum cleaners of a particular model. It costs \$12 to store one vacuum cleaner for one year. There is a fixed cost of \$30 for each order. Find the lot size and the number of orders per year that will minimize inventory costs.

2. **Minimizing inventory costs:** A hardware store sells 96 chainsaws per year. It costs \$5 to store one chainsaw for one year. There is a fixed reordering cost of \$15. Find the lot size and the number of orders per year that will minimize inventory costs.

3. **Minimizing inventory costs:** An art gallery owner expects to sell 90 copies of a limited-edition print during the next year. It costs \$1.50 to store one copy for one year. For each order she places, there is a fixed cost of \$7.50, plus \$0.50 for each copy. Find the lot size and the number of times the gallery owner should order per year to minimize her inventory costs.

4. **Minimizing inventory costs:** The owner of Lamps-4-U expects to sell 180 brass lamps during the year. For each order he places, there is a fixed cost of \$18, plus \$2 for each lamp ordered. It costs \$5 to store one lamp for one year. In what lot size and how many times per year should he reorder to minimize the inventory costs?

5. **Minimizing inventory costs:** A T-shirt company sells 4000 sweatshirts per year. To reorder, there is a fixed cost of \$6 plus \$0.80 for each sweatshirt. It costs \$1.20 to store one sweatshirt for one year. In what lot size and how many times per year should an order be placed to minimize inventory costs?

6. **Minimizing inventory costs:** An office supply store sells 7500 pink highlighters per year. It costs \$0.15 to store one pink highlighter for one year. To reorder these highlighters, there is a fixed cost of \$22.50, plus \$0.10 for each highlighter. In what lot size and how many times per year should an order be placed to minimize inventory costs?

7. **Minimizing inventory costs:** A snowmobile dealer in Minnesota expects to sell 960 snowmobiles during the next year. It costs \$9 to store one snowmobile for one year. To reorder, there is a fixed cost of \$67.50, plus \$7.50 for each snowmobile. In what lot size and how many times per year should an order be placed to minimize inventory costs?

8. **Minimizing inventory costs:** A car dealer expects to sell 1320 cars during the next year. It costs \$660 to store one car for one year. To reorder, there is a fixed cost of \$225, plus \$304 for each car. Find the lot size and the number of orders that should be placed so inventory costs will be minimized.

9. **Maximizing revenue:** A chain of discount stores sells 84 weather radios per month at \$20 each. The owners estimate that for each \$1 increase in price, they will sell 3 fewer radios per month. How much should they charge for their weather radios to maximize their revenue?

10. **Maximizing revenue:** A farmer estimates that if he plants 30 grapefruit trees per acre, the average yield per tree will be 480 pounds. For each additional tree planted per acre, the yield per tree will be reduced by 12 pounds. How many trees should be planted per acre to maximize the yield?

11. **Maximizing revenue:** Sam operates a chain of convenience stores. He estimates that he can sell 600 small packs of gum per day if he charges 75 cents each. Sam determines that for each 10-cent reduction in price, he will sell an additional 80 packs per day. How much should he charge for the small packs of gum to maximize his revenue?

12. **Maximizing revenue:** A sporting goods store sells 200 baseball gloves per month at \$36 each. The owner estimates that for each \$2 increase in price, he will sell 5 fewer gloves. Find the price that will maximize revenue.

13. **Maximizing revenue:** A sports arena has 40 roaming soda salespeople, each of whom sells 200 sodas per event. Management estimates that for each additional salesperson, the yield per salesperson decreases by 4. How many additional salespeople should management hire to maximize the number of sodas sold?

14. **Maximizing revenue:** Ms. Wills owns a 16-rack dry stack boat storage facility. The unit rent per rack is currently \$400 per month, and all racks are rented. Each time rent is increased by \$20, one boat owner will move out. Find the rental price that will maximize Ms. Wills' revenue.

15. **Linear demand function:** A local amusement park found that if the admission was \$7, about 1000 customers per day were admitted. When the admission was dropped to \$6, the park had about 1200 customers per day. Assuming a linear demand function, determine the admission price that will yield maximum revenue.

16. **Linear demand function:** A department store manager has determined that when the price of a tank top was \$12, she sold 100 tank tops per month. However, only 80 tank tops were sold per month when the price was raised to \$14. Assuming a linear demand function, determine the price that would maximize the revenue.

17. **Linear demand function:** The cost of producing x units of an item is $C(x) = 10x + 20$. When the selling price is \$20, twenty-one items are sold. However, when the price is \$16, twenty-three items are sold. Assuming the demand function is linear, determine the price per unit and the number of units sold that will maximize the profit.

18. **Linear demand function:** The manager of a bakery knows he can sell 60 small bags of donut holes when the price is \$1.20 each. If the price is \$1.50, only 48 bags are sold. The total cost function for x bags is $C(x) = 0.70x + 15$ dollars. Assuming a linear demand function, determine the price per bag and the number of bags sold that will maximize the profit.

19. **Linear demand function:** The manufacturer of microwave ovens can sell 800 to his dealers at \$392 each. If the price is \$380, he can sell 1000. The total cost of producing x microwaves is $C(x) = 3600 + 250x - 0.01x^2$ dollars. Assuming the demand function is linear, find the price per microwave and the number of microwaves sold that will maximize profit.

20. **Linear demand function:** A candy store can sell 180 lollipops at 62 cents each. The store can sell 220 lollipops if the price is 54 cents each. The total cost of producing x lollipops is $C(x) = 3050 - 10x + 0.04x^2$ cents. Find the number of lollipops that should be produced to maximize profit.

21. **Profit:** Suppose $P(x)$ represents profit on the sales of x cell phones. Suppose $P(25{,}000) = 12{,}000$, $P'(25{,}000) = 2$, and $P''(25{,}000) = -3$.
    a. Is the company making money or losing it? How much?
    b. If sales are increased, will the profits rise or fall? By how much?
    c. What is the meaning of $P''(25{,}000) = -3$?

22. **Profit:** Suppose the monthly marginal profit from John's online tutoring service is $P'(x) = 3$, where x is the number of subscribers.
    a. The profit function is (choose one): linear, quadratic, or a polynomial of degree 3 or higher.
    b. Suppose monthly costs are $C(x) = 9x + 20$. Assuming initial revenue is 0, what is the revenue function?

23. **Profit:** Suppose the demand for a product is \$12 and the total costs are $C(x) = 0.3x^2 + 2x + 5$.
    a. What is the revenue function?
    b. What is the profit function?
    c. What is the maximum value of the profit?

24. Average cost: $A(x) = \dfrac{C(x)}{x}$ gives average cost.

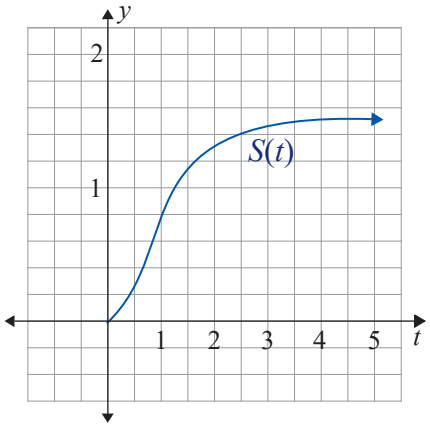
a. Calculate a formula for $A'(x)$.
b. Set $A'(x) = 0$ and solve for $C'(x)$.
c. If average costs are minimal, describe a relationship between average cost and marginal cost. That is, interpret the result of part **b.**

25. Average cost: Suppose that a company's average cost is $A(x) = 0.2x + 3$.
a. Determine the cost function.
b. Determine the marginal average cost.
c. Determine the marginal cost.

26. Earnings: Suppose a company's earnings are given by $E(x) = P(x) + I(x)$, where x is the number of years since 2018, $P(x)$ is the annual profit function, and $I(x)$ is the intangible growth (the growth in value of the company's intangible assets such as its good name). If $P(x) = 1.3x + 2$ and $I(x) = 0.25x + 1$ for a certain company, determine the following.
a. The marginal earnings for year x
b. The actual earnings for 2020
c. The average earnings formula (earnings per year since 2018)

27. Value: The value of a new business franchise grows according to the formula $V(x) = 10 + \dfrac{10x}{1 + 0.5x}$. Here V is in thousands of dollars and x is the number of years after 2012.
a. What is the expected value in 2022?
b. Is the value V increasing or decreasing in 2022?
c. Is the rate of increase in value changing? What has this to do with V' or V''?

28. Value: The timber value of a small stand of pine trees is given by $V(x) = 50\left(1 - \dfrac{1}{x+2}\right)$, where x is the number of years after 2000 and V is in dollars.
a. What was the value in 2000?
b. At what rate was the value changing in 2010?
c. What asymptotes are present and what is their significance in the problem?



WRITING & THINKING

29. Sales function: A sales function $S(t)$ for a new product is shown in the figure. $S(t)$ is total sales (quantity of items) and t is time in months since the product's release. Copy this graph onto your paper and add a graph of a possible $S'(t)$. Locate approximately the point of inflection on your curve for S.

30. Cost function: A certain cost function $C(x)$ satisfies $C(10) = 20$, $C(20) = 40$, and $C(30) = 60$. Suppose $C''(10) = -2$, $C''(20) = 0$, and $C''(30) = 2$. Draw a suitable function.