Testing shows that

$$S''(x) > 0 \quad \text{for} \quad 0 < x < 50$$

and

$$S''(x) < 0 \quad \text{for} \quad 50 < x < 80.$$

Concavity changes from upward on the left side of $S(50)$ to downward on the other, signifying it is a point of diminishing returns.

Thus the point of diminishing returns is at $(50, S(50)) = (50, 5100)$. At the point of diminishing returns, \$50,000 are spent on advertising, and sales in tires are \$5,100,000.

## 12.2 EXERCISES

### PRACTICE

Find both the first and second derivatives for each of the functions in Exercises 1–12. Locate any relative maximum or minimum points and any points of inflection. Determine the intervals on which the function is concave upward or concave downward.

**1.** $f(x) = 7x^2 - 28x + 8$

**2.** $f(x) = 5x^2 - 9x + 2$

**3.** $f(x) = 2x^3 + 5x - 1$

**4.** $f(x) = 3x^3 + 6x - 8$

**5.** $f(x) = x^3 + 2\sqrt{x} + 5$

**6.** $f(x) = x^4 - 3\sqrt{x} + 2$

**7.** $f(x) = (x^2 + 7)^2$

**8.** $f(x) = (2x^2 - 5)^2$

**9.** $f(x) = \sqrt{x^2 + 3}$

**10.** $f(x) = \sqrt[3]{x^2 + 9}$

**11.** $f(x) = \dfrac{3x}{x^2 + 1}$

**12.** $f(x) = \dfrac{2x + 1}{x^2 - 4}$

In Exercises 13–16, find all inflection points. Apply the Second Derivative Test at possible maximum/minimum points. Make a sketch of the graph and confirm your results with a graphing calculator.

**13.** $f(x) = (x + 5)\sqrt[3]{x}$

**14.** $f(x) = (x^2 + 1)\sqrt[3]{x}$

**15.** $f(x) = 2x\sqrt[3]{x + 1}$

**16.** $f(x) = (x + 10)\sqrt[3]{x^2 + 10}$

In Exercises 17–30, use the Second Derivative Test to find all local extrema, if the test applies. Otherwise, use the First Derivative Test.

**17.** $f(x) = x^2 - 3x + 5$

**18.** $f(x) = 8 + 7x - 2x^2$

**19.** $f(x) = x^3 - 3x^2 + 8$

**20.** $f(x) = x^3 + 6x^2 - 10$

**21.** $f(x) = x^3 - 12x + 3$

**22.** $f(x) = x^3 - 3x + 4$

**23.** $f(x) = \dfrac{2}{3}x^3 - x^2 - 4x - 2$

**24.** $f(x) = \dfrac{1}{3}x^3 + x^2 - 3x - 1$

**25.** $f(x) = x^4 - 8x^2 + 7$

**26.** $f(x) = x^4 - 2x^2 + 3$

**27.** $f(x) = x^4 + 2x^3 - 4$

**28.** $f(x) = x^4 - 6x^3 + 8$

**29.** $f(x) = 2x + \frac{8}{x}$

**30.** $f(x) = \frac{x^2 + 9}{x}$

## APPLICATIONS

**31. Point of diminishing returns:** Find the point of diminishing returns for the sales function $S(x) = 112 + 1.8x^2 - 0.1x^3$, where $x$ represents thousands of dollars spent on advertising, $0 \le x \le 10$, and $S$ is sales in thousands of dollars.

**32. Point of diminishing returns:** The sales function for a product is given by $S(x) = 204 + 6.3x^2 - 0.25x^3$, where $x$ represents thousands of dollars spent on advertising, $0 \le x \le 12$, and $S$ is sales in thousands of dollars. Find the point of diminishing returns.

**33. Marginal cost:** The cost function for a particular product is given by $C(x) = 0.1x^3 - 2.4x^2 + 24x + 190$ dollars, where $0 \le x \le 12$. Find the minimum marginal cost.

**34. Marginal cost:** Find the minimum marginal cost of a product if the cost function is given by $C(x) = 0.0001x^3 - 0.036x^2 + 16.8x + 1900$ dollars, where $0 \le x \le 150$.

**35. Law enforcement:** Due to the rapid increase in major crimes, the mayor of a large city plans to organize a major crime task force. It is estimated that for every 1000 persons in the city, the numbers of major crimes will be $N(t) = 56 + 3t^2 - 0.8t^{\frac{5}{2}}$, where $t$ is the number of months after the task force is organized and $0 \le t \le 12$.

**a.** Find the maximum $N(t)$.

**b.** Find the maximum rate of increase in $N(t)$.

**36. Meteorology:** Meteorology records for a certain city suggest that for the month of June, the temperature between midnight and 6:00 p.m. can be approximated by $T(t) = -0.04t^3 + 1.14t^2 - 7.2t + 66$ degrees, where $t$ is the number of hours after midnight and $0 \le t \le 18$.

**a.** Find the maximum and minimum temperatures.

**b.** Find the maximum rate of increase in the temperature.

## WRITING & THINKING

**37.** Given $f(x) = px^3 + bx + 10$, answer the following questions.

**a.** Suppose $p$ and $b$ are positive numbers. What can be said about maximum/minimum points and points of inflection?

**b.** Suppose $p$ and $b$ have opposite signs (one is positive and the other negative). What can be said about maximum/minimum points and points of inflection?

**c.** If the constant term 10 is changed to some other value, do your responses to parts **a.** and **b.** change?

**d.** Put your answers to parts **a.**, **b.**, and **c.** together in a "Lab Report" which discusses the coefficients in the given polynomial $y = px^3 + bx + c$.