$$0=(16)^{\frac{1}{4}}\cdot\frac{3}{4}(81)^{-\frac{1}{4}}\frac{dy}{dt}+(81)^{\frac{3}{4}}\cdot\frac{1}{4}(16)^{-\frac{3}{4}}(4)$$

$$0=\cancel{2}\cdot\frac{\cancel{3}}{{}_{2}\cancel{4}}\cdot\frac{1}{\cancel{3}}\cdot\frac{dy}{dt}+27\cdot\frac{1}{\cancel{4}}\cdot\frac{1}{8}\cdot\cancel{4}$$

$$0=\frac{1}{2}\cdot\frac{dy}{dt}+\frac{27}{8}$$

$$-\left(\frac{1}{2}\cdot\frac{dy}{dt}\right)=\frac{27}{8}$$

$$\frac{dy}{dt}=\left(-\cancel{2}\right)\frac{27}{\cancel{8}_{4}}$$

$$\frac{dy}{dt}=-\frac{27}{4}=-6.75$$

Because the derivative is negative, the capital must **decrease** at a rate of $\frac{27}{4}=6.75$ units per month to keep the current level of production.

11.3 EXERCISES

PRACTICE

Use implicit differentiation to find $\frac{dy}{dx}$ for each of the equations in Exercises 1–20.

1. $2x^2+y^2=4$

2. $x^3+y^3=5$

3. $2x^3+y^3=8$

4. $x^2-y^2=16$

5. $\sqrt{x}+\sqrt{y}=1$

6. $x-\sqrt{y}=2$

7. $x^2y=2$

8. $xy^2=-1$

9. $x^2+xy+y^2=-1$

10. $x^3+2xy-y^2=3$

11. $4x^2+3xy+y^2=2x$

12. $x^3+y^3=3xy$

13. $\frac{1}{x}+\frac{x}{y}=2x$

14. $x^2+\frac{2x}{y}=\frac{1}{x^2}$

15. $x^2+\sqrt{xy}=2y^2$

16. $2y+\sqrt{xy}=5x^2$

17. $x^2y^2+xy^3=x^4$

18. $x^3y+xy^3=3x^3$

19. $x^2+(y-2)^2=16$

20. $x^2+4(y+3)^2=9$

In Exercises 21–30, use implicit differentiation to find $\frac{dy}{dx}$ for the given equations; then find the slope of the tangent line at the given point.

21. $4x^2 - 8y^3 = 24;\ (2, -1)$

22. $3x^3 + 5y^2 + x = 1;\ (-1, 1)$

23. $x^2y - y^2 + 4x + 8 = 0;\ (1, 4)$

24. $5x^2 + xy^2 + 2x = 8;\ (-2, 2)$

25. $x^3 + 2xy - y^2 = 0;\ (3, -3)$

26. $4x^2 - 3xy + y^2 = 7;\ (2, 3)$

27. $\frac{1}{x} + \frac{x}{y^2} = 2x;\ (1, -1)$

28. $3x - \frac{2x}{y^2} + x^2 = 12;\ (3, 1)$

29. $2y + \sqrt{xy} = 5x^2;\ (2, 8)$

30. $x^2 - \sqrt{xy} = 2y^2 + 3x;\ (4, 1)$

In Exercises 31–40, *x* and *y* are functions of a third variable, *t*. Use implicit differentiation to find an expression for $\frac{dy}{dt}$.

31. $x^2 - 4y^2 = 16$

32. $3x^2 + y^4 = 4$

33. $x^3 + 5y^2 = 2x$

34. $6x^2 + 5x = 2y^2$

35. $\sqrt{xy} = 4$

36. $\sqrt{x} + \sqrt{y} = 3$

37. $x^2 + xy + y^2 = 3$

38. $x + 2xy - y^2 = 6$

39. $2xy + x^2 = y^3$

40. $x^2y^2 - y^3 + 4x = 0$

APPLICATIONS

41. Retail sales: The manager of an audio electronics store has determined that the number of stereo receivers and the number of speaker systems sold weekly are related by the equation $0.9y^2 = 10x + xy$, where *x* is the number of receivers and *y* is the number of speaker systems. Find $\frac{dy}{dx}$ if *x* = 12 and *y* = 20, and interpret your answer.

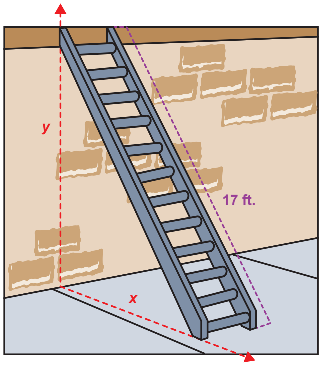
42. Retail sales: The number of pairs of trousers *x* and the number of shirts *y* sold at a department store are related by the equation $36x = 11y + 0.01x^2y$. Find $\frac{dy}{dx}$ when *x* = 10 and *y* = 30, and interpret your answer.

43. Cobb-Douglas production: The level of production of a company is given by $P = 30x^{\frac{1}{3}}y^{\frac{2}{3}}$ units monthly, where *x* is the units of labor and *y* is the units of capital. The company is currently utilizing 64 units of labor and 27 units of capital. If labor is increased by 2 units per week, what will be the change in units of capital per week to maintain the current level of production?

44. Cobb-Douglas production: The level of production of a company is given by $P = 18x^{0.3}y^{0.7}$ units monthly, where *x* is the units of labor and *y* is the units of capital. The company is currently utilizing 35 units of labor and 24 units of capital. If capital is increased by 3 units per week, what will be the change in units of labor per week to maintain the current level of production?

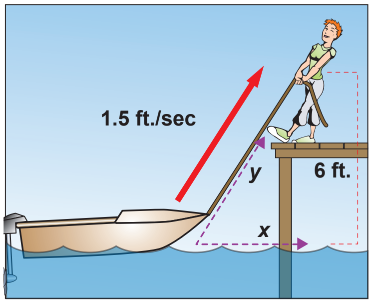
45. **Rate of increase in cost:** The cost of producing x units of a product is given by the function $C(x) = 0.02x^3 - x^2 + 8x + 200$. The factory is currently producing 60 units per week but plans to increase production at a rate of 3 units per week. What will be the rate of increase in the total cost?

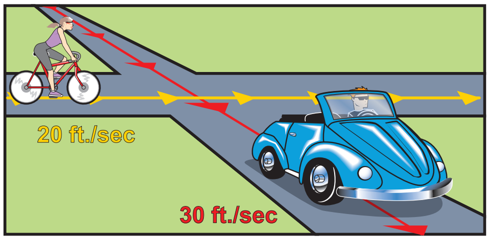
46. **Rate of decrease in cost:** The cost of producing x units of a commodity is given by the function $C(x) = x^2 - 2x^{\frac{3}{2}} + 7x + 180$. Currently, the production level is 36 units per day. The company plans to decrease production at a rate of 2 units per day. What will be the rate of decrease in the total cost?



47. **Sliding ladder:** A 17 ft ladder is leaning against a wall. The bottom of the ladder is pulled away from the wall at a rate of 3 ft/sec. How fast is the top of the ladder moving down the wall when the top is 8 feet above the ground?

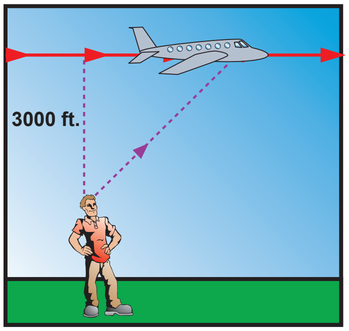
48. **Velocity:** Marijean is standing on a boat dock pulling in her boat by means of a rope attached to a boat at water level. Her hands are 6 feet above the water and she is pulling in the rope at a rate of 1.5 ft/sec. How fast is the boat approaching the dock if there are 10 feet of rope still out?

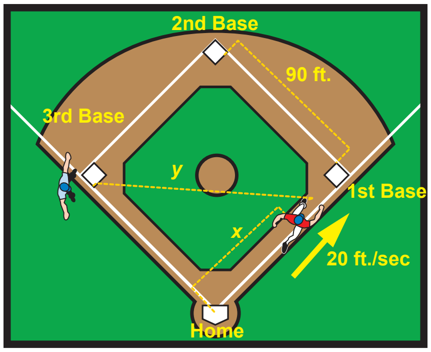




49. **Driving:** A car traveling south at 30 ft/sec crosses an intersection. When the car is 90 feet past the intersection, a bicyclist crosses the intersection traveling east at a rate of 20 ft/sec. How fast is the distance between the car and the bicycle increasing 5 seconds after the bicycle crosses the intersection?

50. **Distance to an airplane:** An airplane traveling at a height of 3000 feet crosses directly over an observer. The speed of the plane is 400 ft/sec. How fast is the distance between the observer and the plane changing after 10 seconds?





51. **Baseball:** A base runner heads towards first base with a speed of 20 ft/sec. A baseball diamond is a square, 90 feet on each side.

a. How long will it take for the runner to reach first base?

b. What is the runner's rate of change of distance from the umpire standing on third base (we will call this $\frac{dy}{dt}$)? (**Hint:** Use the diagram to get a relation between x and y.)

c. What is the runner's speed when he arrives at first base?

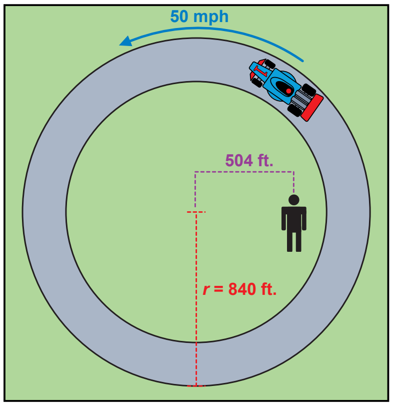
52. **Sailing:** A Coast Guard radar monitoring station on shore observed a sailboat on its radar grid. It was determined that the boat's east-west distance along the shoreline changed by the formula $x = 12 + 0.1t$ and its seaward distance (north-south) changed by the formula $y = 20 - 0.3t$. Here, x and y are in miles and t is in minutes.
   a. What was the sailboat's position at $t = 0$ minutes? Sketch this situation.
   b. Determine $\frac{dx}{dt}$ and $\frac{dy}{dt}$.
   c. What is $\frac{dy}{dx}$ at $t = 10$? Interpret this number.
   d. Where and when will the sailboat hit the shore (assuming it keeps its present heading)?

53. **Bags of oranges:** Suppose the wholesale price in Miami of bags of oranges satisfies a demand equation of $xp + 20p = 1040$, where x is the number of bags supplied and p is the demand (unit price) in dollars.
   a. If 500 bags are available today, what is the unit price?
   b. If the supply is increasing at the rate of 100 bags per day, at what rate is the price changing?

54. **Dripping water:** The radius of a pan of water is 7 cm and water from a tap drips in at the rate of 10 cubic centimeters per minute.
   a. Determine $\frac{dh}{dt}$, where h is the height of water in the pan.
   b. How long will it take to fill the pan to a height of 7 cm?

55. **Demand:** The Arrow Marketing Group sells teddy bears with college logos to college and university gift shops. Their demand equation is $px - 2800 = x$, where x is the quantity of bears and p is the price in dollars that each sells for.
   a. How many bears can be sold (or "demanded") at \$4.50 apiece?
   b. Determine a formula $\frac{dx}{dp}$ and evaluate for p and x as in part a.

56. **Electricity costs:** Electricity costs per semester at Mount State University are calculated by the formula $C = 0.05x + 0.03y - 0.08xy$, where x is a measure of student size and activity, y is dependent on the usage of various buildings and their efficiencies, and C is in millions of dollars.
   a. Determine C if $x = 7$ and $y = 9$.
   b. Determine a general formula for $\frac{dy}{dx}$ and evaluate for $x = 7$ and $y = 9$.



57. **Race track:** A circular race track has a radius of 840 feet. At a certain point in time, t_0, an observer in a maintenance pit 504 feet from the center of the track clocks a car (when it is directly north of him) traveling counterclockwise along the track at a speed of 50 miles per hour from his right to left $\left(\frac{dx}{dt}\right)$. (Use 1 mile = 5280 feet.)
   a. Determine the location of the car on the track. Assume the center of the track is at the origin.
   b. Determine a general equation for $\frac{dy}{dx}$.
   c. What is $\left.\frac{dy}{dx}\right|_{t=t_0}$? Interpret its meaning.
   d. What is $\frac{dy}{dt}$ at $t = t_0$?

58. Chlorination costs: Chlorination costs for the swimming pool at a spa are given by $C = 0.27x + 2y - 0.001xy^2$, where x is the number of weekly swimmers, y is the number of special functions, and C is in dollars.

a. Determine C if $x = 500$ and $y = 4$.

b. Determine a formula for $\frac{dy}{dx}$ and evaluate at $x = 500$ and $y = 4$.

59. Probability measurement: The standard deviation, S, of a binomial random variable is given by $S = \sqrt{np(1-p)}$, where n is the number of trials or repetitions of an experiment and p is the probability of success on one outcome. S is a measure of how the data tends to vary from the center (or mean).

a. For $n = 768$ and $p = 0.25$, determine S.

b. Suppose we consider S as a fixed quantity. Determine $\frac{dp}{dn}$ for n and p as in part a.