Example 5: Marginal Propensity

If the marginal propensity to save is $S'(x) = 0.02x$, what is the marginal propensity to consume?

Solution

$$\begin{aligned} C'(x) &= 1 - S'(x) \\ &= 1 - 0.02x \end{aligned}$$

We replace $S'(x)$ with $0.02x$.

10.9 EXERCISES

PRACTICE

In Exercises 1–10, create an appropriate function of the type indicated.

1. The cost of producing x leather belts is given by $C(x) = 220 + 0.4x$. Determine the average cost function.

2. The cost of manufacturing a certain class of screws for wall hangers is given by $C(x) = 800 + 0.0005x + 0.00002x^2$. Determine the average cost function.

3. A vendor charges \$3 for a hot dog. What is the demand function? What is the revenue function?

4. The Knoll Industrial Supply Company charges for 55-gallon drums using the demand function $p(x) = 22.5 - 0.5x$. What is the revenue function?

5. A supplier of souvenir T-shirts charges street vendors for each order based on a setup fee of \$50 and an item charge of \$1.15 per T-shirt. What is the cost function for the vendor?

6. A supplier of souvenir T-shirts has setup costs of \$25 and shirts cost \$0.40 each. What is the cost function?

7. The answer to Exercise 5 is also a revenue function for the supplier of T-shirts. Using the answers to Exercises 5 and 6, determine a profit function for the supplier of T-shirts.

8. For the supplier in Exercise 6, what is the average cost function?

9. A department store's cost estimate for a line of rocker-recliner chairs is given by $C(x) = 100 + 150x$. The store sells them for \$450 each.
   a. What is the average cost function?
   b. What is the demand function?
   c. What is the revenue function?
   d. What is the profit function?

10. A shoe store estimates a certain line of dress shoes has costs given by $C(x) = 1500 + 30x + 0.05x^2$. The store charges \$75 per pair.
- **a.** What is the average cost function?
- **b.** What is the demand function?
- **c.** What is the revenue function?
- **d.** What is the profit function?

11. Suppose that total national consumption is given by a function $C(x) = 200 - 0.6x - 0.05x^{0.6}$, where x is the total national income.
- **a.** Determine the marginal propensity to consume.
- **b.** Determine the marginal propensity to save.

12. If the marginal propensity to save of a certain country is given by $S'(x) = 0.4x + 0.3$, determine the marginal propensity to consume.

13. The average cost $\overline{C}(x)$ of a product is $\dfrac{C(x)}{x}$, where $C(x)$ is the total cost function.
- **a.** What is the average cost function if its total cost function is $C(x) = 30 + 2x + 0.003x^2$?
- **b.** What is the rate of change of average cost?
- **c.** What value of x results in a minimum average cost?

14. The average cost of a product is given by $A(x) = 20x^{-1} + 3$.
- **a.** Determine the cost function for the product.
- **b.** Determine the marginal cost function.

APPLICATIONS

15. The weekly cost of producing x electric drills is given by the function $C(x) = 2400 + 28x + 0.25x^2$.
- **a.** Find $C(10)$, $C(20)$, and $C(30)$.
- **b.** Find the marginal cost function.
- **c.** Find $C'(10)$, $C'(20)$, and $C'(30)$.
- **d.** Find the average cost function and the marginal average cost function.
- **e.** Find the marginal average cost when $x = 10$, $x = 20$, and $x = 30$.

16. The total cost function for producing x units of a product is given by $C(x) = \dfrac{1}{3}x^3 - \dfrac{1}{2}x^2 + 7x + 18$.
- **a.** Find $C(3)$, $C(4)$, and $C(6)$.
- **b.** Find the marginal cost function.
- **c.** Find $C'(3)$, $C'(4)$, and $C'(6)$.
- **d.** Find the average cost function and the marginal average cost function.
- **e.** Find the marginal average cost when $x = 3$, $x = 4$, and $x = 6$.

17. The total cost of producing x units of a commodity is given by $C(x) = 60 + 10x - 0.5x^2$.
- **a.** Find $C(4)$, $C(6)$, and $C(9)$.
- **b.** Find the marginal cost function.
- **c.** Find $C'(4)$, $C'(6)$, and $C'(9)$.
- **d.** Find the average cost function.
- **e.** Find the marginal average cost when $x = 4$, $x = 6$, and $x = 9$.

18. The total cost of producing x wireless speakers is given by the function $C(x) = 300 + 24x - 0.4x^2 + 0.1x^3$.
a. Find $C(2)$, $C(3)$, and $C(5)$.
b. Find the marginal cost function.
c. Find $C'(2)$, $C'(3)$, and $C'(5)$.
d. Find the average cost function and the marginal average cost function.
e. Find the marginal average cost if $x = 4$.

19. A manufacturer has determined that the revenue from the sale of x cell phones is given by $R(x) = 94x - 0.03x^2$ dollars. The cost of producing x cell phones is $C(x) = 10{,}800 + 34x$ dollars.
a. Find the profit function $P(x)$.
b. Find $P(200)$, $P(400)$, and $P(600)$.
c. Find the marginal profit function $P'(x)$.
d. Find $P'(200)$, $P'(400)$, and $P'(600)$.
e. Find any break-even points.

20. The revenue from the sale of x fire extinguishers is estimated to be $R(x) = 54x - 0.4x^2$ dollars. The total cost of producing x fire extinguishers is $C(x) = 400 + 30x - 0.2x^2$ dollars.
a. Find the profit function $P(x)$.
b. Find $P(20)$, $P(40)$, and $P(60)$.
c. Find the marginal profit function $P'(x)$.
d. Find $P'(20)$, $P'(40)$, and $P'(60)$.
e. Find any break-even points.

21. A company that produces and sells compact refrigerators has found that the revenue from the sale of x refrigerators is $R(x) = 100x - 0.1x^2$ dollars. The cost function is given by $C(x) = 2070 + 25x + 0.1x^2$ dollars.
a. Find the profit function $P(x)$.
b. Find $P(60)$, $P(80)$, and $P(100)$.
c. Find the marginal profit function $P'(x)$.
d. Find $P'(60)$, $P'(80)$, and $P'(100)$.
e. Find any break-even points.

22. A manufacturer has determined that the cost and the revenue of producing and selling x telescopes are $C(x) = x^2 + 20x + 1050$ dollars and $R(x) = 140x - 0.5x^2$ dollars, respectively.
a. Find the profit function $P(x)$.
b. Find $P(30)$, $P(35)$, and $P(40)$.
c. Find the marginal profit function $P'(x)$.
d. Find $P'(30)$, $P'(35)$, and $P'(40)$.
e. Find any break-even points.

23. The owner of a leather craft shop has determined that he can sell x attaché cases if the price is $p = D(x) = 46 + 0.25x$ dollars. The total cost for these cases is $C(x) = 0.15x^2 + 6x + 190$ dollars.
a. Find the revenue function $R(x)$.
b. Find the profit function $P(x)$.
c. Find $P(25)$, $P(30)$, and $P(40)$.
d. Find the marginal profit function $P'(x)$.
e. Find $P'(25)$, $P'(30)$, and $P'(40)$.

24. A firm can sell x items of a product when the price is $p = D(x) = 3.00 - 0.001x$ dollars. The total production costs are $C(x) = 0.002x^2 + 0.72x + 260$ dollars.
- **a.** Find the revenue function $R(x)$.
- **b.** Find the profit function $P(x)$.
- **c.** Find $P(300)$, $P(375)$, and $P(400)$.
- **d.** Find the marginal profit function $P'(x)$.
- **e.** Find $P'(300)$, $P'(375)$, and $P'(400)$.

25. A local publishing company prints a special magazine each month. It has been determined that x magazines can be sold monthly when the price is $p = D(x) = 5.50 - 0.0004x$. The total cost of producing the magazine is $C(x) = 0.0002x^2 + x + 4650$ dollars.
- **a.** Find the revenue function $R(x)$.
- **b.** Find the profit function $P(x)$.
- **c.** Find $P(3000)$, $P(3500)$, and $P(4000)$.
- **d.** Find the marginal profit function $P'(x)$.
- **e.** Find $P'(3000)$, $P'(3500)$, and $P'(4000)$.

26. A sales representative for a company that produces skateboards can sell x units of their deluxe model if the price is $p = D(x) = 79.9 - 0.03x$ dollars. The total cost for these skateboards is given by $C(x) = 0.08x^2 + 5.1x + 5800$ dollars.
- **a.** Find the revenue function $R(x)$.
- **b.** Find the profit function $P(x)$.
- **c.** Find $P(320)$, $P(340)$, and $P(350)$.
- **d.** Find the marginal profit function $P'(x)$.
- **e.** Find $P'(320)$, $P'(340)$, and $P'(350)$.

27. A certain model car has a valuation in dollars given by the formula $f(x) = 12{,}519.3 - 1391.1x$, for $0 \le x \le 7$, where x is the age of the car in years. $x = 0$ corresponds to this calendar year.
- **a.** What is $f(0)$? Interpret this number.
- **b.** What is the marginal valuation? Interpret this number.

28. Based on averaging results at a certain state college, a relationship between grades and SAT scores was found to be $f(s) = 1.36 + 0.00141s$, where s is a student's SAT score and $f(s)$ is the student's graduating GPA (GPA based on 4.0 maximum score).
- **a.** What is the expected GPA for a student with an SAT score of 1000?
- **b.** What is the marginal GPA? Interpret this number.

Exercises 29–31 deal with projections of world population based on estimates of fertility around the world. Use the following background information:

The Total Fertility Rate (TFR) is the average number of children a woman will have. The United Nations projects world population according to assumptions about TFR values. In general, lower values promote economic well-being and lower world population. (**Source:** Population Reference Bureau, "Transitions in World Population," *Population Bulletin*, Vol. 59, No. 1, 36, March 2004.)

29. Total fertility rate: Using a TFR of 1.5, the UN projects total world population, in billions, will be modeled by $F(t)$ where t is the number of years after 2000 and F is the function $F(t) = -\frac{2.1}{10^6}t^4 + \frac{1.6}{10^4}t^3 - 0.00457t^2 + 0.0994t + 6.03$.

a. Give the marginal population function $F'(t)$.
b. Determine the estimate of world population in 2030.
c. What is the marginal population in 2030? Interpret this number.

30. Total fertility rate: Using a TFR of 2.0, the UN projects total world population, in billions, will be modeled by $G(t)$ where t is the number of years after 2000 and G is the function $G(t) = -\frac{2.1}{10^6}t^4 + \frac{1.66}{10^4}t^3 - 0.0045t^2 + 0.113t + 6.03$.

a. Give the marginal population function $G'(t)$.
b. Determine the estimate of world population in 2030.
c. What is the marginal population in 2030? Interpret this number.

31. Total fertility rate: Using a TFR of 2.5, the UN projects total world population, in billions, will be modeled by $H(t)$ where t is the number of years after 2000 and H is the function $H(t) = -\frac{2.6}{10^6}t^4 + \frac{1.87}{10^4}t^3 - 0.00396t^2 + 0.116t + 6.05$.

a. Give the marginal population function $H'(t)$.
b. Determine the estimate of world population in 2030.
c. What is the marginal population in 2030? Interpret this number.

WRITING & THINKING

32. An economics professor claimed that the average cost was a minimum if the average cost equaled the marginal cost. Do you agree? Explain why or why not.

33. "The average revenue $\frac{R(x)}{x}$ is not usually studied in the context of business economics." Argue for or against this statement.