

10.8 EXERCISES

 PRACTICE

Use the various rules of differentiation to find the derivative for each of the functions in Exercises 1–10.

1. $y = x^3 - 7x$

2. $y = 4x^2 - 9x + 2$

3. $y = 0.3x^2 - 4x + 6$

4. $y = 120 + 8x - 0.2x^2$

5. $y = x^3 - 6x^2 + 5x + 2$

6. $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 3x + 4$

7. $y = 2x^{\frac{3}{2}} + 4x^{\frac{1}{2}} - 5$

8. $y = 2x^{-\frac{2}{3}} + 3x^{\frac{1}{3}} + 7$

9. $f(t) = 2t^{-\frac{1}{2}} + t^{\frac{1}{2}} + t$

10. $f(x) = 3x^{-\frac{1}{3}} - 2x^{-\frac{1}{2}} + 1$

In Exercises 11–20, use algebraic techniques to rewrite each function as a sum or difference; then find the derivative.

11. $y = (x+1)(2x-3)$

12. $y = \sqrt{x}(2x^2 + x - 3)$

13. $f(v) = v^{\frac{3}{2}}(v^2 + 2v - 1)$

14. $f(v) = v^{\frac{1}{3}}(6 - 4v + v^2)$

15. $f(x) = \frac{x^4 + 5x^3}{x^2}$

16. $f(x) = \frac{6x^2 + 1}{x^3}$

17. $g(t) = \frac{t-2}{\sqrt{t}}$

18. $g(t) = \frac{t^2 + 3}{\sqrt[3]{t}}$

19. $g(x) = \frac{4x + 5x^{\frac{1}{2}} - 1}{\sqrt{x}}$

20. $g(x) = \frac{3\sqrt{x} + 4x - 2}{x^2}$

In Exercises 21–26, confirm your results with a graphing utility.

21. Let $f(x) = x^3 + 2x - 4$.

- Find the slope of the tangent line at $x = -1$.
- Find the equation of the tangent line at $x = -1$.

22. Let $f(x) = 2x^3 - x^2 - 3x$.

- Find the slope of the tangent line at $x = 2$.
- Find the equation of the tangent line at $x = 2$.

23. Let $g(x) = x^2 + 6x + 5$.

- Find the slopes of the tangent lines at $x = -1$, $x = -3$, and $x = -4$.
- Find the equations of the tangent lines at $x = -1$, $x = -3$, and $x = -4$.
- Sketch the graphs of the curve and the three tangent lines.

24. Let $g(x) = x^2 - 8x + 12$.
- Find the slopes of the tangent lines at $x = 2$, $x = 4$, and $x = 5$.
 - Find the equations of the tangent lines at $x = 2$, $x = 4$, and $x = 5$.
 - Sketch the graphs of the curve and the three tangent lines.
25. Let $f(x) = 8 - x^2$.
- Find the slopes of the tangent lines at $x = -2$, $x = 0$, and $x = 1$.
 - Find the equations of the tangent lines at $x = -2$, $x = 0$, and $x = 1$.
 - Sketch the graphs of the curve and the three tangent lines.
26. Let $f(x) = 10 - 3x - x^2$.
- Find the slopes of the tangent lines at $x = -3$, $x = -2$, and $x = 0$.
 - Find the equations of the tangent lines at $x = -3$, $x = -2$, and $x = 0$.
 - Sketch the graphs of the curve and the three tangent lines.

APPLICATIONS

27. **Velocity of a rocket:** A model rocket is fired vertically upward. The height after t seconds is $s(t) = 192t - 16t^2$ feet.
- Find the velocity at $t = 0$ seconds.
 - Find the velocity at $t = 4$ seconds.
 - When will the velocity be zero?
28. **Velocity of a particle:** A particle moving in a straight line is at a distance of $s(t) = 2.5t^2 + 18t$ feet from its starting point after t seconds, where $0 \leq t \leq 12$.
- Find the velocity at $t = 6$.
 - Find the velocity at $t = 9$.
29. **Population:** A city's population t years from now can be estimated from the formula $P(t) = 9000 + 500t - 72\sqrt{t}$.
- Find the rate at which the city is growing after 4 years.
 - Find the rate at which the city is growing after 9 years.
30. **Cost:** The total cost of producing x units of a product is given by $C(x) = 4000 + 25x - 0.2x^2$ dollars, where $0 \leq x \leq 50$.
- Find the rate of change in the cost when $x = 10$.
 - Find the rate of change in the cost when $x = 30$.
31. **Fuel consumption:** When a factory operates from 6:00 a.m. to 6:00 p.m., its total fuel consumption varies according to the formula $f(t) = 0.9t^2 - 0.3t^{0.5} + 20$, where t is the time in hours after 6:00 a.m. and $f(t)$ is number of barrels of fuel oil.
- How much fuel oil is consumed by noon?
 - What is the rate of consumption of fuel at 10:00 a.m.?
 - What is the average rate of consumption from 6:00 a.m. to 2:00 p.m.?
32. **Population:** The population of bacteria in a lab experiment for BIOL 403 at Nevada Tech is given by $f(x) = 2.2x^{1.5} - 0.7x + 2$, where x is the time in hours after 2:00 p.m. and $f(x)$ is population in suitable units.
- What is the population at 2:00 p.m.?
 - The lab is over at 4:00 p.m. What is the new population of bacteria?
 - What is the average rate of change of bacteria from 2:00 p.m. to 4:00 p.m.?
 - What is the instantaneous rate of change of bacteria at 3:00 p.m.?

- 33. Electrical charge:** The electrical charge on a new cell phone declines according to the formula $C(t) = 15 - 0.1t^2 - 0.5t$, where t is the time in hours following a full charge and $C(t)$ is a measure of the charge.
- To the nearest hour, how long does one have until the charge is fully depleted?
 - What is the instantaneous rate of change, in charge units per hour, at $t = 4$?
 - What is the average rate of change from $t = 0$ to $t = 4$?
- 34. Spreading a rumor:** The number of college students at Salis Technical College who have not heard a new rumor is approximated by the formula $N(x) = 300(1 - 0.004x^2)$, where x is the number of days following the start of a new rumor.
- How many days does it take for 90 percent of the students to hear the rumor?
 - What is the instantaneous rate of change in students after one day? Interpret the meaning of this number.
 - Why is the slope negative in this exercise?
- 35. Birth rate:** The fertility decline in many countries can be modeled by a quadratic equation. In China, from the late 1960s to the present, the number of births per woman has declined according to the formula $f(x) = 0.00675x^2 - 0.3215x + 5.585$, where x is the number of years after 1969. (Source: Nancy E. Riley, "China's Population: New Trends and Challenges," *Population Bulletin*, Vol. 20, No. 2, June 2004.)
- What was the number of births per woman in 1969?
 - What was the number of births per woman in 1999?
 - What was the rate of change of this fertility rate in 1979?
- 36. Birth rate:** In China, from 1964 to the present, the death rate has remained nearly constant at approximately 8 deaths per 1000 people. The yearly birth rate, in births per 1000 people, has declined in most years according to the formula $f(x) = -0.00191x^3 + 0.134x^2 - 3.16x + 44.5$, where x is the number of years since 1964. (Source: Nancy E. Riley, "China's Population: New Trends and Challenges," *Population Bulletin*, Vol. 20, No. 2, June 2004.)
- What number is $f(30)$ and what does it represent?
 - What number is $f'(30)$ and what does it represent?
 - In what year, according to the model, did the number of new births equal the number of deaths?
- 37. Population growth:** The percentage of older persons in China (age 60 and over) has grown since the 1950s according to the formula $f(x) = 0.003727x^2 - 0.105x + 7.063$, where x is the number of years since 1953. (Source: Nancy E. Riley, "China's Population: New Trends and Challenges," *Population Bulletin*, Vol. 20, No. 2, June 2004.)
- What was the percentage of older persons in China in 1953?
 - What is the percentage of older persons projected to be in the year 2025?
 - At what rate was the percentage changing in 2000?

- 38. Population growth:** The population in billions of people in the lesser developed countries has varied according to the formula $P(x) = \frac{4.953}{10^4}x^2 - 0.007352x + 1.7748$, where x is the number of years since 1900. (Source: Population Reference Bureau, "Transition's in World Population," *Population Bulletin*, Vol. 59, No. 1, 5, March 2004.)
- What was the population in 1950?
 - At what rate was the population changing in 1950?
 - What population was projected for 2020 in these countries?
- 39. Death rate:** The death rate in Mexico has varied since 1920 according to the formula $M(x) = -\frac{2.076}{10^4}x^3 + 0.033x^2 - 1.785x + 43.07$, where x is the number of years since 1920. (Source: Population Reference Bureau, "Transition's in World Population," *Population Bulletin*, Vol. 59, No. 1, 5, March 2004.)
- What was the death rate in 1950?
 - At what rate was the death rate changing in 1950?
 - What was the death rate in 2000?
 - In what year did the formula predict that no one would die?
- 40. Birth rate:** The function $f(x) = 0.00375x^2 - 0.2355x + 5.595$ gives the average number of births per woman in Thailand where x denotes the number of years since 1970. (Source: Lori Ashford, "World Population Highlights 2004," BRIDGE Population Reference Bureau, August 2004.)
- What number is $f(10)$ and what does it represent?
 - What number is $f'(10)$ and what does it represent?
- 41. Birth rate:** In Argentina, from 1970 to 2000, the average number of births per woman was given by the function $f(x) = -0.001x^2 + 0.014x + 3.14$, where x is the number of years after 1970. (Source: Lori Ashford, "World Population Highlights 2004," BRIDGE Population Reference Bureau, August 2004.)
- What number is $f(30)$ and what does it represent?
 - What number is $f'(30)$ and what does it represent?