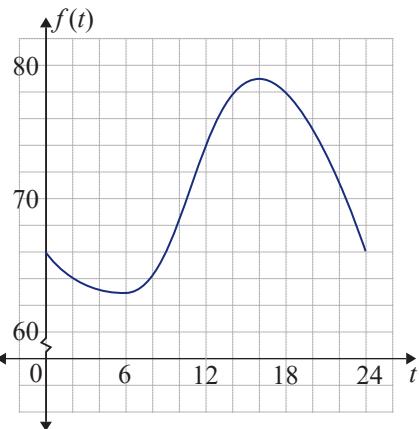


## 10.6 EXERCISES

 PRACTICE

Use the graph to solve Exercises 1–3. The variable  $t$  is the number of hours since midnight and  $f(t)$  is the temperature at time  $t$ .

1. Roughly estimate the instantaneous rate of change of  $f(t)$  at 3:00 p.m. (**Hint:** Extend an imaginary tangent line so as to come close to or to intersect points with integer coordinates.)
2. Estimate the time  $t$  for the lowest temperature  $f(t)$ .
3. Estimate the time  $t$  for the fastest increase in temperature  $f(t)$ .



4. Sketch one graph so that all of the following statements are true.
 

<p>(a) <math>f'(x)</math> is positive for <math>-2 \leq x \leq 6</math>.</p> <p>(c) <math>f'(6) = 0</math></p>	<p>(b) <math>f'(x) &lt; 0</math> for <math>x &gt; 6</math></p> <p>(d) <math>f(6) = 10</math> and <math>f(0) = 1</math></p>
--	--
5. Interpret the meaning of  $f(3) = 14$  and  $f'(3) = 7$  for the function  $f(x) = 2 + x + x^2$ .

 APPLICATIONS

In Exercises 6–15, determine what the slope  $f'$  represents in terms of the subject in the problem.

6.  $f(t)$  is the distance in feet traveled by a car in  $t$  minutes.
7.  $f(s)$  is the total money spent in a department store by  $s$  customers.
8.  $f(n)$  is the number of birds nesting in woods with  $n$  trees per acre.
9.  $f(x)$  is the total cost of manufacturing  $x$  toasters.
10.  $f(u)$  is the total revenue from the sale of  $u$  car radios.
11.  $f(t)$  is the speed of a race car after  $t$  seconds.
12.  $f(v)$  is the total amount of information in bytes fed into a server at Castle Manufacturing Company in  $v$  seconds.
13.  $f(x)$  is the vertical distance in meters traveled by a test rocket in  $x$  seconds.
14.  $f(s)$  is the grade point average of freshmen at Sullivan Technical College where  $s$  is the average SAT score of the freshman class.
15.  $f(t)$  is the cost of calculus books at college bookstores in the United States where  $t$  is the time in years since 1980.

16. Suppose  $f(x)$  is the number of gallons of gas used by a car after it has traveled  $x$  miles.
- Suppose the car gets 20 miles/gallon. What is  $f(100)$ ?
  - Is  $f'(100)$  positive or negative?
  - Would  $f'(100)$  be greater for a subcompact car or for an SUV?
17. Suppose  $f(x)$  denotes the production units for input  $x$  in labor units (man-hours). Suppose  $f(500) = 2000$  and  $f'(500) = 3$ .
- Interpret  $f(500) = 2000$  and  $f'(500) = 3$ .
  - Estimate the increased production if  $x$  is increased from 500 to 501.
18. Suppose  $f(x)$  is the total number of students on a college campus that have the flu and  $x$  is the number of days after the first case is reported. Interpret  $f(8) = 9$  and  $f'(8) = 3$ .
19. Suppose  $f(x)$  is the cost of a Ford Taurus and  $x$  is the age of the car.
- Is  $f'(x)$  positive or negative?
  - Interpret the meaning of  $f'(3) = -2500$ .
20. Average prices for one-bedroom condominiums have steadily risen in Charleston, SC since 2000, according to local reports. Suppose  $f(x) = 3000x + 72,000$  is the cost of a one-bedroom condo and  $x$  is the number of years since 2000.
- Interpret  $f(0) = 72,000$ .
  - Interpret  $f'(x) = 3000$ .
  - Interpret  $f(3) = 81,000$ .
21. Suppose  $f(x)$  denotes the weight of a cancerous tumor  $x$  weeks after discovery. Interpret  $f(3) = 4$  grams and  $f'(3) = 0.4$  grams/week.
22. Water boils at  $212^\circ\text{F}$  and at  $100^\circ\text{C}$ . Water freezes at  $32^\circ\text{F}$  and  $0^\circ\text{C}$ . Let  $F$  denote temperature in degrees Fahrenheit and let  $x$  be temperature in degrees Celsius.
- Write a formula  $F(x) = mx + b$ , which can convert Celsius input  $x$  into Fahrenheit output  $F$ .
  - Use the value of  $m$  to write a formula for  $F'(x)$ .
23. **Birth rate:** The fertility decline in many countries can be modeled by an appropriate equation. In Bangladesh, from 1970 to 2000, patterns of fertility changed according to the equation  $y = -0.11x + 6.45$ , where  $x$  is the time in years beginning in 1970 and  $y$  is the average number of children per woman. (Source: Lori Ashford, "World Population Highlights 2004," BRIDGE Population Reference Bureau, August 2004.)
- What number is  $f(20)$  and what does it represent?
  - What number is  $f'(20)$  and what does it represent?
24. **Birth rate:** Patterns of fertility changed in India from 1970 to 2000 according to the equation  $y = -0.068x + 5.22$ , where  $x$  is the number of years after 1970 and  $y$  is the average number of children per woman. (Source: Lori Ashford, "World Population Highlights 2004," BRIDGE Population Reference Bureau, August 2004.)
- What number is  $f(30)$  and what does it represent?
  - What number is  $f'(30)$  and what does it represent?

- 25. Birth rate:** In China, from 1964 to the present, the death rate has remained nearly constant at about 8 deaths per 1000 persons. However, the yearly birth rate, in births per 1000 people, has declined in most years according to the formula  $f(x) = -0.641x + 35.8$ , where  $x$  is the number of years since 1964. (Source: Nancy E. Riley, “China’s Population: New Trends and Challenges,” *Population Bulletin*, Vol. 20, No. 2, June 2004.)
- What number is  $f(30)$  and what does it represent?
  - What number is  $f'(30)$  and what does it represent?
- (Note: See Exercise 36 in Section 10.8 for a similar problem using a more accurate model than the linear model given here. Both models are based on the same data.)

### TECHNOLOGY

Use a graphing utility in Exercises 26–29 to find the slope of  $f(x)$  at the given point. Sketch the graph of  $f(x)$  and tangent line at the given point on your paper.

- $f(x) = \frac{4+2x}{\sqrt{x}}$ ; (16, 9)
- $f(x) = x^3$ ; (3, 27)
- $f(x) = 2 - 3x + x^2$ ; (1, 0)
- $f(x) = 10^x$ ; (2, 100)
- For the function  $f(x) = 4 - x - 2x^2$ , find a window including the point (0, 4) so that the graph of the function and the tangent at (0, 4) are indistinguishable.
- For the function  $f(x) = x^3$ , make a table with headings  $a$ ,  $f(a)$ , and  $f'(a)$ . Then substitute numbers using  $a = -1, 0, 1, 2, 3, 4$ . Give a formula for  $f'(x)$ .
- Locate (with a graphing utility) the  $x$ - and  $y$ -coordinates of the lowest point on the graph  $f(x) = x^2 - 6x + 11$ . What is the slope at the lowest point?
- For the function  $y = x^2$  add a column to Table 3 to include the  $y$ -intercepts of the tangent line. What curiosity do you observe in the table?
- Find the **LN** button on your graphing calculator and sketch a graph of  $y = \ln x$  on your calculator using the window  $[-2, 8]$  by  $[-0.5, 2]$ . What is the slope at the point (1, 0)? Sketch the graph and tangent on your paper.
- On  $f(x) = \sqrt{x}$ , locate the  $x$ - and  $y$ -coordinates of the point at which the slope is exactly 1. (Hint: Find  $(a, \sqrt{a})$  so that  $f'(a) = 1$ .)
- A bacteria culture in a lab grows according to the formula  $y = 1600(2^t)$  where  $t$  is time in hours and  $y$  is the quantity of bacteria.
  - Interpret the meaning of  $f'(1)$ .
  - Determine  $f''(1)$  using a graphing utility.
- Sketch  $f(x) = (x+10)(x-5)(x-10)$  on a graphing utility. Give the window used.
  - Locate the points  $(a, f(a))$  for which  $f'(a) = 0$ .
  - What is the value of  $f'(2)$ ?

38. Sketch  $y = \frac{1}{x}$  on a graphing utility. Find the  $x$ - and  $y$ -coordinates of any point with slope  $-25$ .
39. Suppose  $f(x) = 2x^2$ . Create a table of values for the slope of  $f'(x)$  (see Table 4). Guess a formula for  $f'(x)$ .