## Example 3: Finding Infinite One-Sided Limits

**a.** Find $\lim_{x\to-3^-}\left(\frac{x+5}{x+3}\right)$.

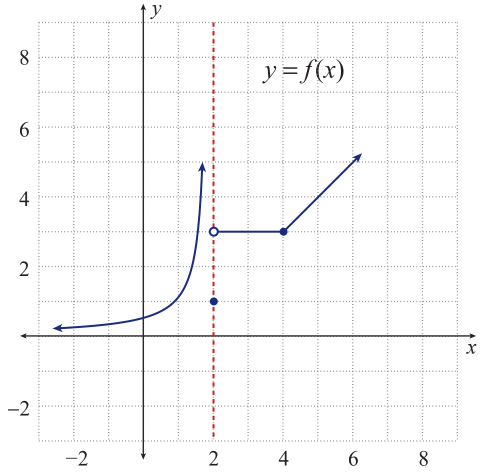
**b.** Find $\lim_{x\to-3^+}\left(\frac{x+5}{x+3}\right)$.

### Solution

**a.** As $x$ approaches $-3$ from the left, the denominator will always be negative but will approach 0; the absolute value of the denominator will get smaller and smaller. For example, $-3.1+3=-0.1$, $-3.01+3=-0.01$, and so on. Meanwhile, the numerator will approach $-3+5=2$. Thus the fraction $\frac{x+5}{x+3}$ will become very large in the negative sense (or unbounded in the negative direction). So $\lim_{x\to-3^-}\left(\frac{x+5}{x+3}\right)=-\infty$.

**b.** Here, as $x$ approaches $-3$ from the right, $x+3$ will always be positive. For example, $-2.9+3=+0.1$, $-2.99+3=+0.01$, and so on. Thus the denominator will approach 0 through positive values, and the numerator will approach $-3+5=2$. Therefore, the fraction $\frac{x+5}{x+3}$ will become unbounded in the positive direction, and we have $\lim_{x\to-3^+}\left(\frac{x+5}{x+3}\right)=+\infty$.

## Example 4: Finding One-Sided Limits Using a Graph




Study the graph shown for $y=f(x)$ and find the following one-sided limits.

**a.** $\lim_{x\to2^-} f(x)$ **b.** $\lim_{x\to2^+} f(x)$ **c.** $\lim_{x\to4^-} f(x)$ **d.** $\lim_{x\to4^+} f(x)$

### Solution

**a.** $\lim_{x\to2^-} f(x)=+\infty$

**b.** $\lim_{x\to2^+} f(x)=3$

**Note:** $f(2)=1$ according to the graph, but this fact does not affect either the left- or right-hand limits in parts **a.** and **b.**

**c.** $\lim_{x\to4^-} f(x)=3$

**d.** $\lim_{x\to4^+} f(x)=3$

# 10.1 EXERCISES

## PRACTICE

In Exercises 1–6, determine the limits. In each case, make a suitable table, with four values, to support your answer. Choose the fourth value ±0.001 from the indicated *a*-value.

**1.** $\lim_{x\to7^-}\left(\frac{x^2-49}{x-7}\right)$

**2.** $\lim_{x\to7^+}\left(\frac{x^2+49}{x-7}\right)$

**3.** $\lim_{x\to 3^+}\left(\frac{x^3-9x^2+27x-27}{x-3}\right)$

**4.** $\lim_{h\to 0^+}\left(\frac{\sqrt{4+h}}{h}\right)$

**5.** $\lim_{a\to 1^+}\left(\frac{a^{10}-1}{a-1}\right)$

**6.** $\lim_{n\to \sqrt{2}^-}\left(\frac{n^2-2}{n-\sqrt{2}}\right)$

Given the table for $\lim_{x\to a^+} f(x)$ in Exercise 7 and $\lim_{x\to a^-} f(x)$ in Exercises 8–9, **a.** give the value for *a* and **b.** determine the limit, if there is one.

**7.**

| $x$ | $y$ |
|---|---|
| 2.500 | 0.2222 |
| 2.100 | 0.2439 |
| 2.010 | 0.2494 |
| 2.001 | 0.2499 |

**8.**

| $x$ | $y$ |
|---|---|
| 3.800 | 15.60 |
| 3.900 | 15.80 |
| 3.990 | 15.98 |
| 3.999 | 15.998 |

**9.**

| $x$ | $y$ |
|---|---|
| 3.000 | 3.43 |
| 3.100 | 11.99 |
| 3.140 | 313.90 |
| 3.141 | 843.60 |

**10.** **a.** Complete the table.

| $x$ | $f(x)=3x-1$ |
|---|---|
| 1 | |
| 1.4 | |
| 1.8 | |
| 1.9 | |
| 1.99 | |
| 1.999 | |

**b.** Find $\lim_{x\to 2^-}(3x-1)$.

**11.** **a.** Complete the table.

| $x$ | $f(x)=x^2-2$ |
|---|---|
| 0 | |
| –0.4 | |
| –0.8 | |
| –0.9 | |
| –0.99 | |
| –0.999 | |

**b.** Find $\lim_{x\to -1^+}(x^2-2)$.

**12.** **a.** Complete the table.

| $x$ | $f(x)=\frac{x^2-1}{x+1}$ |
|---|---|
| 2 | |
| 1.6 | |
| 1.2 | |
| 1.1 | |
| 1.01 | |
| 1.001 | |

**b.** Find $\lim_{x\to 1^+}\left(\frac{x^2-1}{x+1}\right)$.

**13.** **a.** Complete the table.

| $x$ | $f(x)=x^2+3$ |
|---|---|
| 2 | |
| 2.4 | |
| 2.8 | |
| 2.9 | |
| 2.99 | |
| 2.999 | |

**b.** Find $\lim_{x\to 3^-}(x^2+3)$.

**14. a.** Complete the table.

| $x$ | $f(x)=\frac{1}{x-4}$ |
|---|---|
| 3 | |
| 3.4 | |
| 3.8 | |
| 3.9 | |
| 3.99 | |
| 3.999 | |

**b.** Find $\lim_{x\to 4^-}\left(\frac{1}{x-4}\right)$.

**15. a.** Complete the table.

| $x$ | $f(x)=\frac{x}{x+2}$ |
|---|---|
| −3 | |
| −2.6 | |
| −2.2 | |
| −2.1 | |
| −2.01 | |
| −2.001 | |

**b.** Find $\lim_{x\to -2^-}\left(\frac{x}{x+2}\right)$.

**16. a.** Complete the table.

| $x$ | $f(x)=\frac{x^2-4}{x+2}$ |
|---|---|
| −1 | |
| −1.4 | |
| −1.8 | |
| −1.9 | |
| −1.99 | |
| −1.999 | |

**b.** Find $\lim_{x\to -2^+}\left(\frac{x^2-4}{x+2}\right)$.

**17. a.** Complete the table.

| $x$ | $f(x)=\frac{x-3}{x^2-2x-3}$ |
|---|---|
| 4 | |
| 3.6 | |
| 3.2 | |
| 3.1 | |
| 3.01 | |
| 3.001 | |

**b.** Find $\lim_{x\to 3^+}\left(\frac{x-3}{x^2-2x-3}\right)$.

Find In Exercises 18–23, use the graph of $y=f(x)$ to find the limits.

**18.** $\lim_{x\to -1^-} f(x)$

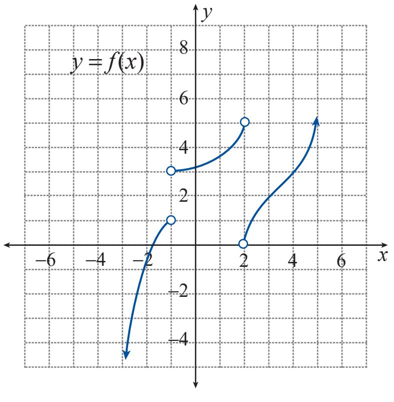
**19.** $\lim_{x\to -1^+} f(x)$

**20.** $\lim_{x\to 2^-} f(x)$

**21.** $\lim_{x\to 2^+} f(x)$

**22.** $\lim_{x\to 3^-} f(x)$

**23.** $\lim_{x\to 3^+} f(x)$



In Exercises 24–29, use the graph of $y = f(x)$ to find the limits.

**24.** $\lim_{x\to -1^-} f(x)$

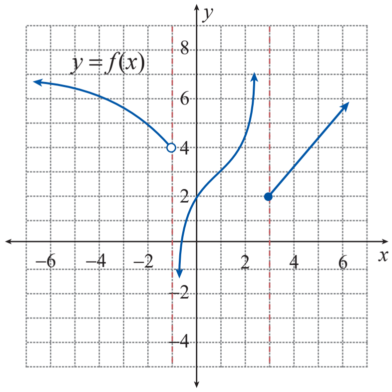
**25.** $\lim_{x\to -1^+} f(x)$

**26.** $\lim_{x\to 0^-} f(x)$

**27.** $\lim_{x\to 0^+} f(x)$

**28.** $\lim_{x\to 3^-} f(x)$

**29.** $\lim_{x\to 3^+} f(x)$




In Exercises 30–35, use the graph of $y = f(x)$ to find the limits.

**30.** $\lim_{x\to 0^-} f(x)$

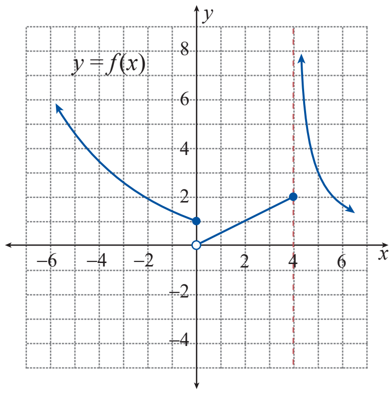
**31.** $\lim_{x\to 0^+} f(x)$

**32.** $\lim_{x\to 4^-} f(x)$

**33.** $\lim_{x\to 4^+} f(x)$

**34.** $\lim_{x\to 2^-} f(x)$

**35.** $\lim_{x\to 2^+} f(x)$




Find the one-sided limits indicated in Exercises 36–59.

**36.** $\lim_{x\to 2^+} (5x-3)$

**37.** $\lim_{x\to -1^+} (2x+7)$

**38.** $\lim_{x\to 0^-} (4-3x)$

**39.** $\lim_{x\to 3^-} (1-6x)$

**40.** $\lim_{x\to 2^-} (x^2-3x+1)$

**41.** $\lim_{x\to -5^+} (x^2+4x-2)$

**42.** $\lim_{x\to -4^+} (x^2-x+3)$

**43.** $\lim_{x\to -3^-} (x^2+2x-3)$

**44.** $\lim_{x\to 10^-} (0.01x^2+7x-30)$

**45.** $\lim_{x\to 10^+} (0.2x^2-5x+6)$

**46.** $\lim_{x\to 0^+} \left(\frac{x-3}{x}\right)$

**47.** $\lim_{x\to 0^-} \left(\frac{2x+1}{x}\right)$

**48.** $\lim_{x\to 1^+} \left(\frac{x-2}{x-1}\right)$

**49.** $\lim_{x\to 1^-} \left(\frac{x-2}{x-1}\right)$

**50.** $\lim_{x\to 2^-} \left(\frac{1}{x+2}\right)$

**51.** $\lim_{x\to 2^+} \left(\frac{1}{x+2}\right)$

**52.** $\lim_{x\to 3^+} \left(\frac{1}{x+1}\right)$

**53.** $\lim_{x\to 1^+} \left(\frac{1}{x-5}\right)$

**54.** $f(x)=\begin{cases}2-3x & \text{if } x<2\\ x-1 & \text{if } x\ge 2\end{cases}$

**a.** $\lim_{x\to 2^-} f(x)$

**b.** $\lim_{x\to 2^+} f(x)$

**55.** $f(x)=\begin{cases}x^2+2 & \text{if } 0\le x\le 3\\ 2x+5 & \text{if } x>3\end{cases}$

**a.** $\lim_{x\to 3^-} f(x)$

**b.** $\lim_{x\to 3^+} f(x)$

**56.** $f(x)=\begin{cases}3x+1 & \text{if } 0\le x\le 4\\ x^2-3 & \text{if } x>4\end{cases}$

**a.** $\lim_{x\to 4^-} f(x)$

**b.** $\lim_{x\to 4^+} f(x)$

**57.** $f(x)=\begin{cases}x^3 & \text{if } x<2\\ x^2+5 & \text{if } x\ge 2\end{cases}$

**a.** $\lim_{x\to 2^-} f(x)$

**b.** $\lim_{x\to 2^+} f(x)$

**58.** $f(x)=\begin{cases}3-2x & \text{if } x<1\\ x & \text{if } 1\le x\le 4\\ \dfrac{1}{x-4} & \text{if } x>4\end{cases}$

**a.** $\lim_{x\to 1^-} f(x)$ **b.** $\lim_{x\to 1^+} f(x)$

**c.** $\lim_{x\to 4^-} f(x)$ **d.** $\lim_{x\to 4^+} f(x)$

**59.** $f(x)=\begin{cases}x^2-1 & \text{if } 0\le x<2\\ 3 & \text{if } 2\le x\le 5\\ \dfrac{1}{x-5} & \text{if } x>5\end{cases}$

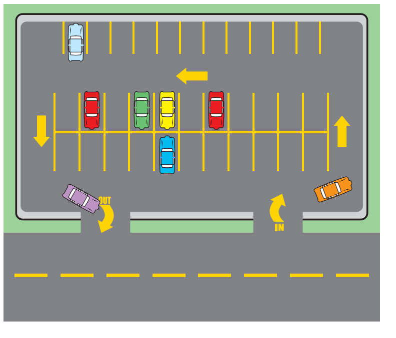
**a.** $\lim_{x\to 2^-} f(x)$ **b.** $\lim_{x\to 2^+} f(x)$

**c.** $\lim_{x\to 5^-} f(x)$ **d.** $\lim_{x\to 5^+} f(x)$

## APPLICATIONS

**60. Parking rates:** The rate for parking in the short-term lot (maximum of 24 hours) at the airport is \$1.00 for the first hour plus \$0.75 for each additional hour or part thereof, with a maximum cost of \$7.00. The function for the cost of parking on this lot for $t$ hours (up to 24 hours) is as follows.

$$C(t)=\begin{cases}1.00 & \text{for } 0<t\le 1\\ 1.75 & \text{for } 1<t\le 2\\ 2.50 & \text{for } 2<t\le 3\\ 3.25 & \text{for } 3<t\le 4\\ 4.00 & \text{for } 4<t\le 5\\ 4.75 & \text{for } 5<t\le 6\\ 5.50 & \text{for } 6<t\le 7\\ 6.25 & \text{for } 7<t\le 8\\ 7.00 & \text{for } 8<t\le 24\end{cases}$$




**a.** Graph the function for $0<t\le 24$ hr.

**b.** Find $\lim_{t\to 3^-} C(t)$.

**c.** Find $\lim_{t\to 3^+} C(t)$.

**d.** Find $\lim_{t\to 8^-} C(t)$.

**e.** Find $\lim_{t\to 8^+} C(t)$.

## WRITING & THINKING

**61.** Suppose $f(x)$ and $g(x)$ are polynomials and $f(t)=0=g(t)=0$ for some $t$. If $\lim_{x\to t^-}\dfrac{f(x)}{g(x)}=L$, must $\lim_{x\to t^+}\dfrac{f(x)}{g(x)}$ also be $L$?