Example 4: Equations with Positive Rational Exponents

Solve the following equations with rational exponents.

a. $x^{\frac{3}{4}} - 8 = 0$

Solution

$$\begin{aligned} x^{\frac{3}{4}} - 8 &= 0 \\ x^{\frac{3}{4}} &= 8 \\ \sqrt[4]{x^3} &= 8 \\ x^3 &= 8^4 \\ x &= 8^{\frac{4}{3}} \\ x &= \left(8^{\frac{1}{3}}\right)^4 \\ x &= (2)^4 \\ x &= 16 \end{aligned}$$

Since the term containing the rational exponent can be rewritten as a radical expression, we will begin by isolating that term.

Raising both sides to the fourth power eliminates the fourth root.

Raising both sides to the $\frac{1}{3}$ power solves the equation for x, but we can evaluate the expression on the right-hand side.

Verify that this number solves the original equation.

b. $\left(18x^2 - 54x - 8\right)^{\frac{1}{6}} = 2$

Solution

$$\begin{aligned} \left(18x^2 - 54x - 8\right)^{\frac{1}{6}} &= 2 \\ \sqrt[6]{18x^2 - 54x - 8} &= 2 \\ 18x^2 - 54x - 8 &= 2^6 \\ 18x^2 - 54x - 8 &= 64 \\ 18x^2 - 54x - 72 &= 0 \\ 18\left(x^2 - 3x - 4\right) &= 0 \\ x^2 - 3x - 4 &= 0 \\ (x-4)(x+1) &= 0 \\ x &= 4, -1 \end{aligned}$$

The exponent of $\frac{1}{6}$ indicates we should raise both sides to the sixth power in order to eliminate the radical.

We are left with a second-degree polynomial equation that can be solved by factoring.

Note that both solutions solve the original equation.

1.6 EXERCISES

PRACTICE

State any restrictions on x, then solve the proportions. See Example 1.

1. $\frac{4x}{7} = \frac{x+5}{3}$

2. $\frac{3x+1}{4} = \frac{2x+1}{3}$

3. $\frac{10}{x} = \frac{5}{x-2}$

4. $\frac{8}{x-3} = \frac{12}{2x-3}$

5. $\frac{4}{x-4}=\frac{2}{x+3}$

6. $\frac{3}{x+5}=\frac{6}{x-2}$

7. $\frac{x+2}{5x}=\frac{x-6}{3x}$

8. $\frac{x-4}{3x}=\frac{x-2}{5x}$

9. $\frac{5x+2}{x-6}=\frac{11}{4}$

10. $\frac{x+9}{3x+2}=\frac{5}{8}$

State any restrictions on *x*, and then solve the equations. See Example 2.

11. $\frac{5x}{4}-\frac{1}{2}=-\frac{3}{16}$

12. $\frac{x}{6}-\frac{1}{42}=\frac{1}{7}$

13. $\frac{3x-1}{6}-\frac{x+3}{4}=\frac{7}{12}$

14. $\frac{x-2}{3}-\frac{x-3}{5}=\frac{13}{15}$

15. $\frac{2+x}{4}-\frac{5x-2}{12}=\frac{8-2x}{5}$

16. $\frac{4x+1}{5}=\frac{2x+3}{2}-\frac{x+2}{4}$

17. $\frac{2}{3x}=\frac{1}{4}-\frac{1}{6x}$

18. $\frac{1}{x}-\frac{8}{21}=\frac{3}{7x}$

19. $\frac{3}{5x}-\frac{1}{5}=\frac{3}{4x}$

20. $\frac{3}{8x}-\frac{7}{10}=\frac{1}{5x}$

21. $\frac{3}{4x}-\frac{1}{2}=\frac{7}{8x}+\frac{1}{6}$

22. $\frac{5}{3x}+\frac{1}{2}=\frac{7}{9x}-\frac{5}{6}$

23. $\frac{2}{4x+1}=\frac{4}{x^2+9x}$

24. $\frac{3}{4x-1}=\frac{4}{x^2+x}$

25. $\frac{9}{x^2-6x}=\frac{5}{2x-3}$

26. $\frac{-9}{x^2+5x}=\frac{8}{4-9x}$

27. $\frac{x}{x-4}-\frac{4}{2x-1}=1$

28. $\frac{x}{x+3}+\frac{1}{x+2}=1$

29. $\frac{x+2}{x+1}+\frac{x+2}{x+4}=2$

30. $\frac{3x-2}{x+4}+\frac{2x+5}{x-1}=5$

31. $\frac{2}{4x-1}+\frac{1}{x+1}=\frac{3}{x+1}$

32. $\frac{x-2}{x+4}-\frac{3}{2x+1}=\frac{x-7}{x+4}$

33. $\frac{x-2}{x-3}+\frac{x-3}{x-2}=\frac{2x^2}{x^2-5x+6}$

34. $\frac{x}{x-4}-\frac{12x}{x^2+x-20}=\frac{x-1}{x+5}$

35. $\frac{3x+5}{3x+2}+\frac{8x+16}{3x^2-4x-4}=\frac{x+2}{x-2}$

36. $\frac{3x+5}{3x+2}-\frac{4-2x}{3x^2+8x+4}=\frac{x+4}{x+2}$

37. $\frac{3}{3x-1}+\frac{1}{x+1}=\frac{4}{2x-1}$

38. $\frac{2}{x+1}+\frac{4}{2x-3}=\frac{4}{x-5}$

Solve the following radical equations. See Example 3.

39. $\sqrt{4-x}-x=2$

40. $\sqrt{x^2-4x+5}-x+2=0$

41. $\sqrt{x^2-4x+4}+2=3x$

42. $\sqrt{50+7s}-s=8$

43. $\sqrt[4]{2x+3}=-1$

44. $\sqrt{11x+3}+4x=18$

45. $\sqrt{x+10}+1=x-1$

46. $\sqrt{x+1}+10=x-1$

47. $\sqrt{x^2-10}-1=x+1$

48. $\sqrt[3]{5x^2-14x}=-2$

49. $\sqrt{4z+41}+3=z+2$

50. $\sqrt[3]{3-2x}-\sqrt[3]{x+1}=0$

51. $\sqrt[4]{x^2-x}=\sqrt[4]{x-1}$

52. $\sqrt[5]{7t^2+2t}=\sqrt[5]{5t^2+4}$

53. $\sqrt[3]{y^3-7y+2}=\sqrt[3]{2-3y}$

54. $\sqrt{3y+4}+\sqrt{5y+6}=2$

55. $\sqrt{3-3x}-3=\sqrt{3x+2}$

56. $\sqrt{2b-1}+3=\sqrt{10b-6}$

57. $\sqrt{5x+5}=\sqrt{4x-7}+2$

58. $\sqrt{14y^2-18y+4}+2=2y$

59. $\sqrt{9x+4}=\sqrt{7x+1}+1$

Solve the following equations. See Example 4.

60. $(x+3)^{\frac{1}{4}}+2=0$

61. $(2x-5)^{\frac{1}{4}}=(x-1)^{\frac{1}{4}}$

62. $(2x-1)^{\frac{2}{3}}=x^{\frac{1}{3}}$

63. $(3y^2+9y-5)^{\frac{1}{2}}=y+3$

64. $(3x-5)^{\frac{1}{5}}=(x+1)^{\frac{1}{5}}$

65. $w^{\frac{3}{5}}+8=0$

66. $z^{\frac{4}{3}}-\frac{16}{81}=0$

67. $x^{\frac{2}{3}}-\frac{25}{49}=0$

68. $(x-2)^{\frac{2}{3}}=(14-x)^{\frac{1}{3}}$

69. $(y-2)^{\frac{2}{3}}=(13y-66)^{\frac{1}{3}}$

70. $(x^2+21)^{\frac{-3}{2}}=\frac{1}{125}$

71. $(x^2+7)^{\frac{-3}{2}}=\frac{1}{64}$

APPLICATIONS

72. Computers: Making a statistical analysis, Ana found 3 defective computers in a sample of 20 computers. If this ratio is consistent, how many defective computers does she expect to find in a batch of 2400 computers?

73. Manufacturing: At the Bright-As-Day light bulb plant, 3 out of each 100 bulbs produced are defective. If the daily production is 4800 bulbs, how many are defective?

74. Education: The University of Arizona has a ratio of 1 professor for every 23 students. If there are 1600 faculty members at the university, how many students are enrolled there?

75. Baseball: New York Yankees player Didi Gregorius has a recorded batting average of 15 hits for every 50 times at bat. If he maintains this average, how many at bats will he need to achieve 111 hits? (Round to the nearest whole number.)

76. Cartography: On a map of Maryland, one inch represents 4 miles. If there are 8.5 inches between Baltimore, MD and Washington, DC, how far are the two cities from each other?

77. Architecture: A floor plan is drawn to scale in which 1 inch represents 4 feet. What size will the drawing be for a room that is 30 feet by 40 feet? (**Hint:** Set up two proportions.)

78. Baking: The recipe for Nestle Tollhouse Chocolate Chip Cookies calls for 2 cups of chocolate chips to make 5 dozen cookies. If you want to bake 17 dozen cookies, how many cups of chocolate chips do you need?

79. Car maintenance: In the instructions for Never-Ice Antifreeze it states that 4 quarts of antifreeze are needed for every 10 quarts of radiator capacity. If Sal's car has a 22-quart radiator, how many quarts of antifreeze will it need?

80. Landscape architecture: An architect is to draw plans for a city park. He intends to use a scale of $\frac{1}{2}$ inch to represent 25 feet. How many inches will be needed to use for the length and width of a rectangular playing field that is 50 yards by 125 yards? (1 yard = 3 feet)

81. Testing cars: A test driver wants to increase the speed of the car he is driving by 3 miles per hour every 2 seconds. But he can only check his speed every 5 seconds because he is busy with other items during the test drive.
a. By how much should he increase his speed in 5 seconds?
b. If he starts checking his speed at 40 miles per hour, how fast should he be going after 10 seconds?

82. Decorating: Jack and Diane are decorating a nursery room for their baby, which will be born in a few months. In one hour, Jack can get $\frac{1}{6}$ of the nursery done and Diane can get $\frac{1}{12}$ of the nursery done. If they work together, they can get $\frac{1}{x}$ of the nursery done in one hour. Determine how many hours it will take Jack and Diane to decorate the nursery if they work together by solving the equation $\frac{1}{6}+\frac{1}{12}=\frac{1}{x}$ for x.

83. Printing: A local print shop has a big order of pamphlets to print, so they decide to use two of their printers for the one job. The newer printer can print the pamphlets four times as fast as the older printer. That means in one hour, the newer printer can complete $\frac{1}{x}$ of the print job and the older printer can complete $\frac{1}{4x}$ of the print job. Working together, the printers can complete the job in 4 hours. Determine how many hours it would take the newer printer to print all of the pamphlets by itself by solving the equation $\frac{1}{x}+\frac{1}{4x}=\frac{1}{4}$ for x.

84. Construction: Two groups of civil engineers are surveying an area to prepare for the construction of a shopping center. The first group is full of new college graduates and it will take them four more hours than it takes the second group, which is full of seasoned professionals. The second group can complete the job in x hours. This means that in one hour, the first group can complete $\frac{1}{x+4}$ of the job and the second group can complete $\frac{1}{x}$ of the job. Working together, they can complete the surveying job in $\frac{15}{4}$ hours. Determine how many hours it would take each team to complete the job individually by solving the equation $\frac{1}{x+4}+\frac{1}{x}=\frac{4}{15}$ for x.

85. Running: Terrence and Alicia are competing in a marathon where the average running speed is x kilometers per hour. Terrence is running 2 kilometers per hour slower than the average running speed. Alicia is running 2 kilometers per hour faster than the average running speed. After a certain amount of time, Terrence ran 4 kilometers and Alicia ran 6 kilometers.

a. Determine the speed of the average runner by solving the equation $\frac{4}{x-2}=\frac{6}{x+2}$ for x.
b. What was Terrence's average running speed?
c. What was Alicia's average running speed?
d. How long did it take Terrence to run 4 kilometers and Alicia to run 6 kilometers?

WRITING & THINKING

In simplifying rational expressions, the result is a rational or polynomial expression. However, in solving equations with rational expressions, the goal is to find a value (or values) for the variable that will make the equation a true statement. Many students confuse these two ideas. To avoid confusing the techniques for adding and subtracting rational expressions with the techniques for solving equations, simplify the expression in part **a**. and solve the equation in part **b**. Explain, in your own words, the differences in your procedures. Assume no denominator has a value of 0.

86. a. $\frac{10}{x}+\frac{31}{x-1}+\frac{4x}{x-1}$

b. $\frac{10}{x}+\frac{31}{x-1}=\frac{4x}{x-1}$

87. a. $\frac{-4}{x^2-16}+\frac{x}{2x+8}-\frac{1}{4}$

b. $\frac{-4}{x^2-16}+\frac{x}{2x+8}=\frac{1}{4}$

88. a. $\frac{3x}{x^2-4}+\frac{5}{x+2}+\frac{2}{x-2}$

b. $\frac{3x}{x^2-4}+\frac{5}{x+2}=\frac{2}{x-2}$

89. a. $\frac{7}{5x}+\frac{2}{x-4}-\frac{3}{5x}$

b. $\frac{7}{5x}+\frac{2}{x-4}=\frac{3}{5x}$

90. a. $\frac{2}{x+9}-\frac{2}{x-9}+\frac{1}{2}$

b. $\frac{2}{x+9}-\frac{2}{x-9}=\frac{1}{2}$