c. $[3, 4) \cap (4, 9) = \varnothing$ — These two intervals have no elements in common, so their intersection is the empty set.

d. $(-\infty, 4] \cup (-1, \infty) = (-\infty, \infty)$ — The union of these two intervals is the entire set of real numbers.

Example 8: Union and Intersection

Simplify each of the following set expressions.

a. $\{1, 2\} \cup \{0, 3\}$

b. $\{x, y, z\} \cap \{w, x\}$

c. $\mathbb{Z} \cup \mathbb{R}$

d. $\mathbb{Z} \cap \mathbb{R}$

Solutions

a. The union of the two sets consists of all elements in either set: $\{0, 1, 2, 3\}$.

b. The intersection consists only of elements in both sets: $\{x\}$.

c. Since the integers are all also real numbers, the union of these two sets is simply the set of real numbers $\mathbb{R}$. We say that $\mathbb{Z}$ is *contained* in $\mathbb{R}$.

d. Similarly, since all integers are also real numbers, the integers are the elements contained in both sets. Thus, the intersection is $\mathbb{Z}$.

0.2 EXERCISES

PRACTICE

Identify the components of the algebraic expressions, as indicated. See Example 1.

1. Identify the terms in the expression $3x^2y^3 - 2\sqrt{x+y} + 7z$.
2. Identify the coefficients in the expression $3x^2y^3 - 2\sqrt{x+y} + 7z$.
3. Identify the factors in the term $-2\sqrt{x+y}$.
4. Identify the terms in the expression $x^2 + 8.5x - 14y^3$.
5. Identify the coefficients in the expression $x^2 + 8.5x - 14y^3$.
6. Identify the factors in the term $8.5x$.
7. Identify the terms in the expression $\dfrac{-5x}{2yz} - 8x^5y^3 + 6.9z$.
8. Identify the coefficients in the expression $\dfrac{-5x}{2yz} - 8x^5y^3 + 6.9z$.
9. Identify the factors in the term $\dfrac{-5x}{2yz}$.

Evaluate the following algebraic expressions for the given values of the variables. See Example 2.

10. $3x^3 + 5x - 2$ for $x = -3$

11. $-8(2x - y) + 4x^2$ for $x = 3$ and $y = 4$

12. $\sqrt{2x} + \dfrac{3x}{4}$ for $x = 8$

13. $3x^2y^3 - 2\sqrt{x+y} + 7z$ for $x = -1, y = 2,$ and $z = -2$

14. $-3\pi y + 8x + y^3$ for $x = 2$ and $y = -2$

15. $\dfrac{|x|\sqrt{2}}{x^3y^2} - \dfrac{3y}{x}$ for $x = -3$ and $y = 2$

16. $y\sqrt{x^3 - 2} + \sqrt{x - 2y} - 3y$ for $x = 3$ and $y = -\dfrac{1}{2}$

17. $\left| -x^2 + 2xy - y^2 \right|$ for $x = -3$ and $y = -5$

18. $\dfrac{1}{32}x^2y^3 + y\sqrt{x} - 7y$ for $x = 4$ and $y = 2$

19. $6x^2 + 3\pi y + y^2$ for $x = 3$ and $y = 2$

20. $|x - 9y| - (8z - 8)$ for $x = -3, y = 1,$ and $z = 5$

21. $\dfrac{x^2y^3}{8z} - \dfrac{|2xy|}{8z}$ for $x = 2, y = -1,$ and $z = 3$

22. $5\sqrt{x+6} - 8y^2$ for $x = 10$ and $y = -2$

Identify the property that justifies each of the following statements. If one of the cancellation properties is being used to transform an equation, identify the quantity that is being added to both sides or the quantity by which both sides are being multiplied. See Examples 3 and 5.

23. $(x - y)(z^2) = (z^2)(x - y)$

24. $3 - 7 = -7 + 3$

25. $(3x + 2) + z = 3x + (2 + z)$

26. $4(y - 3) = 4y - 12$

27. $-3(4x^6z) = (-3)(4)(x^6z) = -12x^6z$

28. $4 + (-3 + x) = (4 - 3) + x = 1 + x$

29. $-2(4 - x) = -8 + 2x$

30. $(x + y)\left(\dfrac{1}{x+y}\right) = 1$

31. $(-5 + 1)(7^7) = (7^7)(-5 + 1)$

32. $-5(-7x^8y^4z) = [(-5)(-7)](x^8y^4z)$

33. $25x^3 = 10y \Leftrightarrow 5x^3 = 2y$

34. $-14y = 7 \Leftrightarrow y = -\dfrac{1}{2}$

35. $14 - x = 2x \Leftrightarrow 14 = 3x$

36. $5 + 3x - y = 2x - y \Leftrightarrow 5 + x = 0$

37. $x^2z = 0 \Rightarrow x^2 = 0$ or $z = 0$

38. $(a + b)(x) = 0 \Rightarrow a + b = 0$ or $x = 0$

39. $\dfrac{x}{6} + \dfrac{y}{3} - 2 = 0 \Leftrightarrow x + 2y - 12 = 0$

40. $(x - 3)(x + 2) = 0 \Rightarrow x - 3 = 0$ or $x + 2 = 0$

41. $21x^4 = 15y^4z \Leftrightarrow 7x^4 = 5y^4z$

42. $6x + \frac{25}{4}y^9 - z = \frac{1}{4}y^9 - z \Leftrightarrow 6x + 6y^9 = 0$

Evaluate each of the following expressions. Be sure to use the correct order of operations. See Example 6.

43. $2 + 3 - 4 \div 8 + (-1)^2$

44. $\frac{-2\left(13 - \sqrt{9} + 2\right)}{14 - 4 \div 2}$

45. $-3^2 - 2 \div 2$

46. $\left(-3^2 - 2\right) \div 2$

47. $\frac{\sqrt{\sqrt{81} + 4^2}}{10(4 - 7 \div 2)}$

48. $4\pi + 6^{\sqrt{5 - \frac{2}{2}}} - 3\pi\left[8 - 15 \div (2 + 3)\right]$

49. $4 - 10 \cdot (-1) \div 5 + (-8)^2$

50. $-3^2 + 2 \cdot \sqrt{2 + 1 \cdot 2} - 7\pi$

51. $1 \div 6 + 3^{\sqrt{2^2}} - (-4 \cdot 2)$

52. $\frac{8 - 9 \cdot 5 - 7}{-4(-9 - 5 \div (2 + 4))}$

53. $-3 + 6 \cdot 1 \div 5 + (-3)^3$

54. $-5^2 + 4 \cdot \sqrt{2 + 7 \cdot 2} - 2\pi$

55. $9 \div 2 + 2^{\sqrt{2^4}} - (1 \cdot 2)$

56. $\frac{4 + 3 \cdot 8 - 6}{-5(3 - 8 \div (2 + 5))}$

Use a calculator to evaluate each of the following expressions. Be sure to use the correct order of operations. Round your answers to two decimal places. See Example 6.

57. $(-3.28)^2 + 4 \cdot \sqrt{2 + 7 \cdot 3} - 2\pi$

58. $2.66 - 7 \cdot 4 \div 5 + (2 \div 3)^2$

59. $\frac{7.6 - 5.2 \cdot 9.8 - 8.1}{-3.22(11 - 6 \div (-1.45 + 6.32))}$

60. $7 \div 4.6 + 2.4^{\sqrt{3}} - (1.23 \cdot 2)^4$

Translate each of the following directions into an algebraic expression.

61. Begin with 3. Add 7, and multiply the result by 3. Subtract 5. Take the square root, raise the result to the 3rd power, and then multiply by $-\frac{1}{5}$.

62. Begin with –6. Add 4, raise the result to the 3rd power, multiply by –2, and take the fourth root of the result.

63. Begin with x. Subtract 4, and take the third root of the result. Divide by 2, and square the result.

Simplify the following set expressions. See Examples 7 and 8.

64. $[-7, 7) \cup (2, 5)$

65. $(-5, 2] \cup (2, 4]$

66. $(-5, 2] \cap (2, 4]$

67. $[3, 5] \cap [2, 4]$

68. $(-\infty, 4] \cup (0, \infty)$

69. $(-\infty, \infty) \cap [-\pi, 21)$

70. $[2, \infty) \cap (-4, 7) \cap (-3, 2]$

71. $(3, 5] \cup [5, 9]$

72. $[-\pi, 2\pi) \cap [0, 4\pi]$

73. $\mathbb{Q} \cap \mathbb{Z}$

74. $\mathbb{N} \cup \mathbb{R}$

75. $\mathbb{N} \cup \mathbb{Z} \cap \mathbb{Q}$

76. $(-4.8, -3.5) \cap \mathbb{Z}$

APPLICATIONS

77. At the beginning of the month, your checking account contains \$128. For your birthday, your mother deposits \$50 and your grandmother deposits \$25. After you write three checks for \$17, \$23, and \$62, you make a deposit of \$41. At the end of the month, your bank removes half of the balance to put in your savings account and then charges you a \$5 fee for doing so. How much do you have remaining in your checking account?

78. A particular liquid boils at 268 °F. Given the formula $C = \frac{5}{9}(F - 32)$ for converting temperatures from Celsius (C) to Fahrenheit (F), find the boiling point of this liquid in the Celsius scale. Round your answer to two decimal places.

79. Stephen received \$75 as a gift from his aunt. With this money, he decided to start saving to buy the newest gaming console, which costs \$398 after tax. After working two weeks at his part-time job, he got one check for \$123 and a second check for \$98. How much more does Stephen need to save to buy his gaming console?

80. Body mass index, abbreviated BMI, is one way doctors determine an adult's weight status. A BMI below 18.5 is considered underweight, the range 18.5–24.9 is normal, the range 25.0–29.9 is overweight, and a BMI above 30.0 indicates obesity. The formula used to determine BMI is $\text{BMI} = 703\left(\frac{\text{weight in pounds}}{(\text{height in inches})^2}\right)$. Derek weighs 180 lbs and is 73 inches tall. Use this formula to determine Derek's BMI and weight status. Round your answer to one decimal place.

81. The Du Bois Method provides a formula used to estimate your body's surface area in meters squared: $\text{BSA} = 0.007184h^{0.725}w^{0.425}$, where h is height in centimeters and w is weight in kilograms. Assume Juan is 193 cm tall and weighs 88 kg. Use the Du Bois Method to estimate his body's surface area in square meters. Round your answer to two decimal places.

82. Samantha drops a tennis ball from the top of the mathematics building. If it takes the ball 3.42 seconds to hit the ground, use the formula $\text{distance} = \frac{1}{2}(\text{acceleration})(\text{time})^2$ to find the height of the building, which is equivalent to the distance the ball falls. Use the value of $32\,\text{ft/s}^2$ for the acceleration of a falling object. Round your answer to the nearest foot.

WRITING & THINKING

83. Choose a number. Multiply it by 3 and then add 4. Now multiply by 2 and subtract 8. Finally divide by 6. What do you notice about your final answer? Explain why you got this as a result.

84. Use your knowledge of the order of operations to check the following problem for accuracy. Explain any errors you find.

$$\begin{aligned} -8 \div 4 + 2^3 - (3 \cdot 2) &= -8 \div 4 + 2^3 - (6) \\ &= -8 \div 4 + 8 - 6 \\ &= -8 \div 4 + 2 \\ &= -8 \div 6 \\ &= \frac{-4}{3} \end{aligned}$$

85. A mnemonic is a device used to recall particular information. For example, "*My Very Educated Mother Just Served Us Nachos*" is often used to recall the order of the planets in our solar system: *My* = Mercury, *Very* = Venus, *Educated* = Earth, and so on. Come up with your own mnemonic for remembering the order of operations.

86. After taking a poll in her town, Sally began grouping the citizens into various sets. One set contained all the citizens with brown hair and another set contained all the citizens with blue eyes. What do you know about the citizens who would be listed in the union of these two sets? What do you know about the citizens who would be listed in the intersection of these two sets?

87. In your own words, explain the difference between a union and an intersection of two sets.

TECHNOLOGY

Use a graphing utility to evaluate the following algebraic expressions.

88. $\sqrt{x^4 y - z} + \dfrac{x - y^3}{z^2}$ for $x = -3$, $y = 2$, and $z = -2$

89. $\dfrac{\left(x - pq^2\right)^3}{2q^3}$ for $x = -5$, $p = 2$, and $q = -3$

90. $\dfrac{\left|x^2 - y^3\right| - 4x}{3y^5}$ for $x = 2$ and $y = 3$

91. $\sqrt{p^3 q - q^3} - \left|p + q^2\right|$ for $p = -5$ and $q = 2$