CHAPTER 6 PROJECT

Balancing Nutrition in a Clinical Dietetics Study

You are assisting a registered dietitian at a university hospital who is designing meal plans for patients with specific dietary requirements. Each patient needs precise amounts of protein, carbohydrates, and fat from a limited set of food sources. In this project, you will use systems of linear equations, matrix operations, Gauss-Jordan elimination, inverse matrices, and Cramer's Rule to formulate meal plans and analyze nutrient interactions.

1. A post-surgery patient has a doctor's recommendation of 40 g of protein and 60 g of carbohydrates from two food sources. Food A provides 8 g of protein and 10 g of carbohydrates per serving. Food B provides 5 g of protein and 15 g of carbohydrates per serving. Set up a system of two equations with two unknowns (the servings of A and B) for achieving the doctor's recommendation, and solve it by substitution.

2. A diabetic patient's meal ideally contains exactly 35 g of protein, 50 g of carbohydrates, and 20 g of fat. Three food sources are available: Food X (10 g of protein, 5 g of carbohydrates, and 4 g of fat per serving), Food Y (5 g of protein, 20 g of carbohydrates, and 2 g of fat per serving), and Food Z (3 g of protein, 10 g of carbohydrates, and 6 g of fat per serving). Write the 3×3 system for this meal plan, and solve it using elimination.

3. Rewrite the system from Question 2 as an augmented matrix, and solve it using Gauss-Jordan elimination (reduced row echelon form). Show each row operation and verify the solution.

4. The dietitian tracks three patients' daily intake (in grams of protein, carbohydrates, and fat for each patient) over two days using matrices. The results are as follows.

$$\text{Day 1: } D_1 = \begin{bmatrix} 42 & 58 & 22 \\ 38 & 65 & 18 \\ 45 & 52 & 25 \end{bmatrix}, \quad \text{Day 2: } D_2 = \begin{bmatrix} 40 & 62 & 20 \\ 36 & 60 & 22 \\ 48 & 55 & 24 \end{bmatrix}$$

Compute $D_1 + D_2$ (the two-day totals) and $\frac{1}{2}(D_1 + D_2)$ (the two-day averages). Which patient had the highest average protein intake?

5. Each gram of protein provides 4 calories, each gram of carbohydrates provides 4 calories, and each gram of fat provides 9 calories. Expressing the calorie conversion as a 3×1 column matrix $K = \begin{bmatrix} 4 \\ 4 \\ 9 \end{bmatrix}$, compute D_1K to find each patient's total caloric intake on Day 1. Which patient consumed the most calories?

6. The dietitian wants to quickly solve meal plans for different patients using the same three foods (X, Y, and Z) from Question 2. The coefficient matrix is as follows.

$$A = \begin{bmatrix} 10 & 5 & 3 \\ 5 & 20 & 10 \\ 4 & 2 & 6 \end{bmatrix}$$

Find A^{-1} using Gauss-Jordan elimination on the augmented matrix $[A \mid I]$. Then use A^{-1} to solve for a new patient who needs 28 g of protein, 55 g of carbohydrates, and 18 g of fat.

7. A pediatric patient requires exactly 24 g of protein, 36 g of carbohydrates, and 14 g of fat. Using the same coefficient matrix A from Question 6, apply Cramer's Rule to find the servings for each. Show the determinant of A and at least one replacement determinant in full.

8. The hospital kitchen operates as part of an interconnected system. Three departments (Kitchen, Purchasing, and Cleaning) each consume some output from the others. The input-output matrix is as follows.

$$A = \begin{bmatrix} 0.10 & 0.15 & 0.05 \\ 0.20 & 0.05 & 0.10 \\ 0.10 & 0.10 & 0.05 \end{bmatrix}$$

The external demand matrix (patient meals, supply orders, and sanitation services) is $D = \begin{bmatrix} 100 \\ 80 \\ 60 \end{bmatrix}$ (in staff-hours). Set up and solve the Leontief equation $\left(I - A\right)X = D$ to find the total staff-hours each department must work.

9. The dietitian designs a second meal plan in addition to the one in Question 2, targeting the same nutritional goals: 35 g of protein, 50 g of carbohydrates, and 20 g of fat. In the new plan, only food sources *X* and *Y* are to be used.

Plan 1: Uses Foods *X*, *Y*, and *Z* (from Question 2)

Plan 2: Uses Foods *X* and *Y* only (no *Z*)

The two-food plan requires solving only the protein and carbohydrate equations (a 2×2 system). Solve it for the stated goals and determine if this two-food solution also meets the fat requirement. If not, calculate the fat shortfall. What does this tell you about the importance of having at least as many food options as nutritional constraints?

10. The dietitian asks, "Is it always possible to meet exact nutritional targets with a given set of foods?" Using what you learned in this project, explain when a system of nutritional equations has a unique solution, infinitely many solutions, or no solution. Give a real-world interpretation of each case. Under what conditions would the determinant of the coefficient matrix be zero, and what would that mean for the meal plan? Finally, explain why the inverse matrix approach (Question 6) is especially valuable in a hospital setting where the same foods are available but patient needs change daily.