CHAPTER 6 PROJECT

Managing Operations at a Regional Distribution Center

You are a logistics intern at a regional distribution center that supplies products to retail stores across three states. The center manages shipments, allocates warehouse space, and tracks costs across multiple product categories. You will use systems of linear equations (substitution, elimination, Gauss-Jordan), determinants and Cramer's Rule, basic matrix operations, inverse matrices, and Leontief input-output analysis to solve operational problems.

1. The center has two types of trucks designated for shipping their products: electronics (x) and furniture (y), and a truck will carry only one product type. Last week, the center dispatched 50 trucks carrying a combined total of 1200 items. Each electronics truck carries 30 items, and each furniture truck carries 20 items. Assuming each truck is loaded to capacity, set up a system of equations for the number of each type used and solve the system by substitution. How many trucks of each type were used?

2. This week, the center adds a third category: clothing (z). The constraints are $x + y + z = 60$ trucks total, $30x + 20y + 40z = 1700$ items shipped, and the labor-hours equation is $5x + 8y + 3z = 350$. Solve this 3×3 system using elimination. Show all steps clearly. What does the solution obtained when changing the last equation to $5x + 8y + 3z = 310$ suggest about this new scenario?

3. Rewrite the system from Question 2 as an augmented matrix. Use Gauss-Jordan elimination to reduce it to reduced row echelon form. Verify that your solution matches the one obtained by elimination.

4. The center tracks weekly costs across three regions using the following matrix.

$$C = \begin{bmatrix} 12{,}000 & 8500 & 5200 \\ 11{,}400 & 9100 & 4800 \\ 13{,}200 & 7800 & 600 \end{bmatrix}$$

Each row is a week (Weeks 1–3), and each column is a region (East, Central, West). Calculate the total cost for each region over the three weeks. Then calculate the average weekly cost for each region.

5. The center receives a matrix of unit shipping costs and a matrix of shipment volumes, as follows.

Unit costs ($ per item): $U = \begin{bmatrix} 2.50 & 3.00 \\ 1.80 & 2.20 \end{bmatrix}$

Volumes (items shipped): $V = \begin{bmatrix} 400 & 250 \\ 600 & 350 \end{bmatrix}$

Rows represent product types (electronics, furniture), and columns represent regions (East, West). Compute the matrix product UV. What does each entry in the resulting matrix represent?

6. The center's manager wants to solve the pricing system $AX = B$, where:

$$A = \begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 26 \\ 46 \end{bmatrix}$$

Find the inverse A^{-1} using the formula for a 2×2 matrix. Then compute $X = A^{-1}B$. Verify by substituting back into the original equations.

7. The distribution center participates in a simple regional economy with three sectors: Manufacturing (M), Transportation (T), and Retail (R). The input-output matrix is:

$$A = \begin{bmatrix} 0.20 & 0.10 & 0.05 \\ 0.15 & 0.10 & 0.10 \\ 0.10 & 0.20 & 0.15 \end{bmatrix}$$

External demand is $D = \begin{bmatrix} 50 \\ 40 \\ 60 \end{bmatrix}$ (in millions of dollars). Set up the Leontief equation $(I - A)X = D$. Write out $(I - A)$ explicitly.

8. Using the Leontief equation from Question 7, solve for $X = (I - A)^{-1} D$. Find the inverse of $(I - A)$ using Gauss-Jordan elimination or a calculator, and multiply by D. What total output must each sector produce to satisfy both internal and external demand?

9. The center's shipment allocation problem can be modeled as $AX = B$, where

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 3 & 1 \\ 1 & 2 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 22 \\ 30 \\ 31 \end{bmatrix}.$$

a. Compute the determinant of A using cofactor expansion along the first row. Show your work.

b. Use Cramer's Rule to solve for x_1, x_2, and x_3. Show the determinant calculation for at least one variable.

c. Verify your solution by substituting back into all three original equations.

10. Your supervisor states, "We should always use Cramer's Rule for solving systems because it gives exact answers." Evaluate this claim. Cramer's Rule does give exact answers, but is it always the best method? Compare Cramer's Rule, Gauss-Jordan elimination, and inverse matrices in terms of efficiency, scalability, and ease of use. Under what circumstances would you recommend each method? Consider systems of size 2×2, 3×3, and larger. What happens to the number of determinant calculations as the system grows?