CHAPTER 4 PROJECT

Bacterial Growth and Radioactive Decay in a Biology Lab

You are a lab assistant in your university's biology department. This semester, the lab is running two parallel projects: studying bacterial colony growth for a microbiology course and calibrating radioactive tracer decay rates for a medical imaging research study. You will use exponential functions, logarithmic functions, and their properties to model growth, decay, half-lives, pH measurements, and carbon dating.

1. A bacterial culture starts with 500 cells and doubles every 3 hours. Write the population as a function of time: $P(t) = 500 \cdot 2^{\frac{t}{3}}$, where t is in hours. Identify the initial value and the base. Evaluate $P(6), P(12)$, and $P(24)$. How many cells are present after one full day?

2. The lab also expresses the same growth using the continuous model $P(t) = 500e^{kt}$. Using the fact that the culture doubles in 3 hours, find k by solving $1000 = 500e^{3k}$. Show the logarithmic steps. Then verify that both models give the same population at $t = 6$ hours.

3. The lab's radioactive tracer, Iodine-131, has a half-life of 8 days. A sample starts at 120 milligrams. The decay function is given by $A(t) = 120\left(\frac{1}{2}\right)^{\frac{t}{8}}$. Evaluate $A(8), A(16)$, and $A(32)$. How much of the tracer remains after 32 days? Why is the decay constant positive?

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4. Rewrite the Iodine-131 decay function in continuous exponential form, $A(t) = 120e^{kt}$, by finding k from the half-life condition. Compare this value of k to the decay constant you found in Question 2. What is the key difference? What does the sign of k tell you about the biological or physical process being modeled?

5. The research team can safely use at most 15 mg of Iodine-131 for a usable medical scan. Starting from the 120 mg sample, how many days until the tracer decays to exactly 15 mg? Set up and solve the equation using logarithms. Show every step.

6. The bacteria culture from Question 1 is growing in a petri dish that can support a maximum of 1,000,000 cells. Using the continuous model $P(t) = 500e^{0.231t}$, how many hours until the population reaches this carrying capacity? Explain why this prediction is likely too optimistic: What happens to bacterial growth as the population approaches the dish's limits?

7. The pH of a solution is defined by $\text{pH} = -\log\left[\text{H}^+\right]$, where $\left[\text{H}^+\right]$ is the hydrogen ion concentration in moles per liter. The lab prepares three solutions for the bacteria experiment:

Solution A: $\left[\text{H}^+\right] = 10^{-4}$

Solution B: $\left[\text{H}^+\right] = 3.2 \times 10^{-7}$

Solution C: $\left[\text{H}^+\right] = 10^{-9}$

Calculate the pH of each solution. Which is most acidic? Which is most basic? The bacteria thrive in a pH range of 6.5 to 7.5. Which solution(s) are suitable for the culture?

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8. A different bacterial strain has a growth rate that depends on incubation temperature:

At 25 °C: $P_1(t) = 200e^{0.15t}$

At 37 °C: $P_2(t) = 200e^{0.35t}$

How long does each population take to reach 10,000 cells? Solve both equations. Calculate the ratio of the two times. The lab manual states that "raising the temperature by 12 °C roughly doubles the growth rate." Do your calculations support this claim?

9. Carbon-14 dating uses the decay model $A(t) = A_0e^{-0.000121t}$, where t is in years. An archaeology class brings a bone fragment to your lab. Testing reveals it retains 35% of its original Carbon-14. Estimate the age of the bone by solving $0.35A_0 = A_0e^{-0.000121t}$. Show all logarithmic steps. The professor suggests the bone could be from a 2000-year-old Roman settlement. Is the Carbon-14 evidence consistent with this claim?

10. The exponential model $P(t) = 500e^{0.231t}$ predicts that the bacterial population will reach 1 billion cells within about 63 hours. Explain why this is unrealistic. What biological factors prevent unlimited exponential growth? Describe the general shape of a more realistic growth curve and name the type of function that models it. Finally, consider: if exponential models always break down eventually, why do biologists still use them? Under what conditions is the exponential model a useful approximation?

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