CHAPTER 4 PROJECT

Analyzing Financial Products at a Local Bank

You have just started a summer internship at a local bank branch. Your manager asks you to analyze several of the bank's financial products, including savings accounts, certificates of deposit (CDs), and auto loans. You will use exponential and logarithmic functions to compare products, project account growth, estimate depreciation, and help the bank communicate clearly with clients.

1. The bank offers a savings account that earns 3.5% annual interest compounded yearly. A client deposits $5000. Write the account balance as an exponential function: $A(t) = 5000(1.035)^t$. Identify the initial value, the base, and whether the function represents growth or decay. Evaluate $A(5)$ and $A(10)$.

2. A different savings product earns 2.8% annual interest compounded quarterly. Using the formula $A(t) = P\left(1+\frac{r}{n}\right)^{nt}$ with $P = \$5000$, $r = 0.028$, and $n = 4$, write the balance function. Calculate the balance after 5 years and compare it to the 3.5% annual product from Question 1 at the same time.

3. The bank also offers a continuously compounded CD at 3.2% annual interest. The balance follows $A(t) = Pe^{rt}$. For a $5000 deposit, calculate the balance after 5 years and after 10 years. Compare all three products (Questions 1, 2, and this one) at the 5-year mark. Which earns the most, and by how much?

4. A client asks: "How long until my money doubles? " A quick way to estimate doubling time for an investment is the Rule of 72, which says you can divide 72 by the annual interest rate (as a percent) to get an approximate number of years. Use the rule to estimate the doubling time for the 3.2% annual product and compare it to the exact doubling time for a 3.2% continuously compounded CD by solving $2 = e^{0.032t}$ using logarithms. How close is the Rule of 72 estimate?

5. A client deposits \$8000 into the continuously compounded CD at 3.2%. She needs the account to reach \$12,000 for a car down payment. Set up and solve the equation $12,000 = 8000e^{0.032t}$ for *t*. Show all logarithmic steps. How many years will she need to wait?

6. The bank uses a depreciation model for auto loans. A car purchased for \$28,000 depreciates according to $V(t) = 28,000(0.85)^t$, where *t* is years after purchase. How much is the car worth after 3 years? After how many years will the car be worth less than \$10,000? Set up and solve the inequality using logarithms.

7. Your manager asks you to compare two CD offers from competing banks:

Bank A: 4.1% compounded monthly

Bank B: 4.0% compounded continuously

For a \$10,000 deposit held for 7 years, which bank offers the higher balance? Calculate the effective annual rate (EAR) for each. Which rate should clients compare when shopping for CDs, and why?

8. A client is deciding between paying off a \$6000 credit card balance at 19.9% APR compounded monthly or investing that same \$6000 in a CD at 4.5% compounded monthly. If no payments or withdrawals are made for 3 years, what will the credit card balance grow to? What will the CD be worth? What is the net financial difference? Explain why comparing the two growth rates side by side is more informative than looking at either one alone.

9. Sound intensity is measured in decibels using $L = 10 \cdot \log\left(\frac{I}{I_0}\right)$, where $I_0 = 10^{-12}$ watts/m^2. The bank's lobby has a sound level of 60 dB during busy hours. A renovation would reduce the level to 50 dB. Calculate the actual sound intensity I for each level. By what factor does the intensity decrease? Explain why a 10 dB change represents a much larger change in actual intensity than it appears on the decibel scale.

10. At the end of your internship, the branch manager asks: "Should we tell clients to focus on the interest rate or the compounding frequency when choosing a savings product?" Use your results from Questions 1 through 3 and Question 7 to support your answer. Then address a broader question: the bank's marketing materials claim that continuous compounding "makes your money grow infinitely faster." Is this claim accurate? Use specific numbers from your calculations to evaluate it. What would you recommend the bank say instead?