CHAPTER 3 PROJECT

Studying Water Quality in a Campus Watershed

You are a research assistant for an environmental science team monitoring the water quality of a stream that flows through your university's campus. The team tracks dissolved oxygen levels, pollutant concentrations, bacteria counts, and temperature across seasons. You will use function notation, domain and range, quadratic models, polynomial and rational functions, transformations, and rational inequalities to analyze the data and advise the university's sustainability office.

1. The dissolved oxygen level in the stream (mg/L) depends on water temperature (°C) and is modeled by $D(T) = -0.2T + 14$, valid for water temperatures between 5 °C and 30 °C. State the domain and range of D using interval notation. Evaluate $D(10)$ and $D(25)$. Which temperature produces a higher dissolved oxygen level? Why does this make biological sense?

2. The daily bacteria count (in thousands per mL) in the stream after a rainfall event is modeled by $B(t) = 3t^2 - 24t + 85$, where t is days since the rainfall. Evaluate $B(0), B(4)$, and $B(8)$. Identify this as a quadratic function and state whether the parabola opens upward or downward.

3. Find the vertex of $B(t) = 3t^2 - 24t + 85$ and interpret both coordinates. The state water quality standard requires bacteria counts below 50,000 per mL (below 50 in the model's units). Does the stream ever meet this standard during the first 10 days after rainfall? Show the inequality and solve.

4. The concentration of a chemical pollutant downstream from a campus parking lot is modeled by:

$$P(d) = \frac{120}{d+3}$$

where d is distance downstream in meters and P is concentration in mg/L. Evaluate $P(0), P(7)$, and $P(27)$. What happens to P as d gets very large? Identify the horizontal asymptote and explain what it means for the stream's ability to dilute pollutants over distance.

5. The environmental team defines 'unsafe' pollutant levels as concentrations above 10 mg/L. Using $P(d) = \frac{120}{d+3}$, set up and solve the inequality $P(d) > 10$ to find the distances over which the water is unsafe. Interpret your answer for the sustainability office.

6. The stream's water temperature (°C) over a 12-month period is modeled by:

$$T(m) = -2(m-7)^2 + 26$$

where m is the month ($m = 1$ is January). Identify this as a transformation of the parent function $f(m) = m^2$. Describe each transformation (reflection, shifts, stretch). Find the peak temperature and the month it occurs.

7. Seasonal nitrate levels (mg/L) follow the polynomial model:

$$N(m) = 0.1m^3 - 2.1m^2 + 12m + 5, \text{ for } m = 1 \text{ through } m = 12$$

Evaluate $N(m)$ for $m = 1, 3, 5, 7, 9$, and 12. During which months are nitrate levels above 15 mg/L? Describe the seasonal pattern and suggest an environmental explanation.

8. The rate at which the stream naturally processes pollutants (mg removed per hour) is modeled by:

$$R(c) = \frac{50c}{c+8}$$

where c is the current pollutant concentration in mg/L. Evaluate $R(2), R(8), R(40)$, and $R(200)$. What value does R approach as c increases without bound? Explain what this biological ceiling means: Why can't the stream process pollutants faster than this limit, no matter how polluted the water becomes?

9. After a sewage leak, the team compares two models for E. coli counts (thousands per mL, t in hours):

Model A: $E_A(t) = -5t^2 + 40t + 100$

Model B: $E_B(t) = \dfrac{800}{t+2}$

Evaluate both at $t = 0$, 4, 8, and 16. Which model predicts that E. coli eventually reaches zero (or negative)? Which predicts a persistent low-level presence? Which behavior is more biologically realistic, and why?

10. The sustainability office asks for your overall assessment. You have used linear, quadratic, polynomial, and rational functions across your analyses. Which type of function was most useful for modeling short-term events (like a pollutant spike after rainfall)? Which was most useful for modeling long-term or distance-based processes (like dilution downstream)? Identify one place in your analysis where the model clearly breaks down. What does this teach you about the relationship between mathematical models and natural systems?