CHAPTER 1 PROJECT

Tracking a Flu Outbreak on Campus

The campus health center has asked your math class to help analyze data from a flu outbreak that swept through the university last semester. The center tracked the number of infected students, monitored recovery rates, and collected survey data on preventive behaviors. Your task is to use linear equations and inequalities, quadratic equations, and rational equations to interpret the data and make recommendations for future flu seasons. All models in this project are based on simplified versions of the health center's actual data.

1. During the first two weeks of the outbreak, the number of new flu cases reported each day increased at an approximately constant rate. To describe this early trend, the health center used the linear model $C = 12t + 5$, where t is the number of days since the first reported case and C is the number of new flu cases on day t. What does the coefficient 12 represent in this context? What does the number 5 represent?

2. Use the model $C = 12t + 5$ to estimate the number of new flu cases reported on day 3 and on day 10. Based on this model, determine the first day on which the predicted number of new cases exceeds 100.

3. To monitor the spread of illness, the campus health center tracked demand for flu testing. During the early stage of flu season, daily testing demand is modeled by $D = 10t + 15$, where t is the number of days since testing began and D is the number of students requesting flu tests on day t. The health center can administer up to 120 flu tests per day.

a. On which day does the demand for testing first exceed the health center's daily capacity?

b. To address rising demand, the university hires a temporary testing nurse, increasing daily testing capacity by 30 tests. Using the model, determine the first day on which testing demand exceeds this expanded capacity.

4. As part of its analysis, the campus health center examined the overall pattern of cumulative flu cases during the early stage of the outbreak. Using a simplified model based on cumulative data, the total number of cases after t days is approximated by $T = 6t^2 + 5t + 5$, where t is the number of days since the outbreak began. How many total flu cases had occurred by the end of day 7? By the end of day 14? What is the difference between these two totals, and what does this difference represent in the context of the outbreak?

5. After the initial linear phase, the daily number of new flu cases began to change as the outbreak progressed. To model daily new cases over a longer time period, the campus health center used the quadratic equation $N = -0.3t^2 + 12t + 5$, where t is the number of days since the outbreak began and N represents the number of new cases reported on day t.

a. Use the quadratic formula to find the values of t for which the model predicts zero new cases. Show all steps.

b. In your own words, explain what it means for this model to produce a value of zero.

6. The campus health center uses the equation $D = \dfrac{1200}{2c + 10}$ to model how long its supply of antiviral medication will last, where c represents the daily number of flu cases and D is the number of days until the supply runs out. A pharmacist wants to reorder supplies before the medication supply drops below 6 days. Solve the equation $D = 6$ to determine the daily case count at which this occurs. Show all work and verify your solution.

7. During peak flu season, the campus health center estimates that the average total time a patient spends in the health center (including waiting and being seen) depends on the number of nurses on duty. This relationship is modeled by $A = \dfrac{30 + 3n}{n}$, where n is the number of nurses on duty and A is the average total time per patient, in minutes. The health center wants to reduce the average patient time to 5 minutes. How many nurses are needed to meet this goal? Interpret your answer in the context of staffing during a flu outbreak.

8. From Question 7, the health center determined the number of nurses needed to keep the average patient time at 5 minutes. However, staffing decisions are also limited by cost. Suppose each nurse on duty costs the health center $320 per day, and the health center has a daily staffing budget of $4800 during peak flu season.

a. Write an inequality that represents the staffing budget constraint.

b. Determine the maximum number of nurses the health center can afford under this budget.

c. Based on your result from Question 7, does the staffing budget allow the health center to meet its goal of a 5-minute average patient time? Explain.

9. The health center has two options for distributing flu prevention kits:

Option 1: Campus mail, with a fixed cost of $2500 plus $3.50 per kit

Option 2: Student volunteers, with a fixed cost of $800 plus $6.00 per kit

a. Write a cost equation for each option, where C is the total cost and k is the number of kits distributed.

b. Find the number of kits for which the total cost of both options is the same.

c. Using the solution from part (b), explain how the health center should decide between the two distribution options when distributing fewer kits versus more kits.

10. The campus health center is preparing a report for the university administration and asks for your assessment of the models used to analyze the flu outbreak: the linear model for daily new cases $C = 12t + 5$ and the quadratic model for daily new cases $N = -0.3t^2 + 12t + 5$. Compare these two models by addressing the following: Under what conditions is each model most appropriate? Which model better describes the overall progression of the outbreak, and why? Identify features of a real flu outbreak (such as those observed during COVID-19) that are not accurately captured by either model.