CHAPTER 14 PROJECT

Modeling the Spread of an Infectious Disease

You are a research assistant in your university's epidemiology department. The department is modeling an outbreak of a respiratory illness in a city of 200,000 people. In this project, you will use indefinite integrals, Riemann sums, the Fundamental Theorem of Calculus, area between curves, and differential equations to analyze infection rates, cumulative case counts, recovery dynamics, and the effectiveness of interventions.

1. The daily rate of new infections (in cases per day) during the first phase of the outbreak is

$$I'(t) = 120 + 18t - 0.5t^2,$$

where t is the number of days since the outbreak began. Find the cumulative infection function $I(t)$ by evaluating the indefinite integral of $I'(t)$. Given that $I(0) = 0$ (there are no cumulative cases at $t = 0$), determine the constant of integration.

2. Using your result from Question 1, calculate the total number of infections during the first 14 days by evaluating $I(14)$. Then compute the number of infections that occurred from day 7 to day 14, given by $I(14) - I(7)$. What fraction of the total infections during the first two weeks occurred in the second week?

3. Use a left-endpoint Riemann sum with $n = 7$ subntervals (each of width 2 days) to approximate $\int_0^{14} I'(t)\,dt$. Compare your approximation to the exact value from Question 2. Does the Riemann sum overestimate or underestimate the true value? Explain your reasoning based on the behavior of $I'(t)$.

4. The recovery rate (in patients per day) is modeled by $R'(t) = 80e^{0.04t}$. Find the total number of recoveries over the first 20 days by evaluating a definite integral. Use substitution if needed.

5. The number of active cases is given by $A(t) = I(t) - R(t)$, where $R(t) = \int R'(t)\,dt$. Find $R(t)$, given that $R(0) = 0$. Write an expression for $A(t)$ and then evaluate $A(14)$. How many people are actively infected on day 14?

6. Two intervention strategies are being analyzed. Under Strategy 1 (masking mandate), the daily rate of new infections is $I_1'(t) = 100e^{-0.05t}$. Under Strategy 2 (quarantine), the infection rate is $I_2'(t) = 150e^{-0.10t}$.

a. Compute the total number of new infections under each strategy over 30 days.

b. Which strategy results in fewer total infections?

c. Determine the day at which the two infection rates are equal. Then compute the area between the two rate functions from $t = 0$ to that day and interpret the area. Which strategy results in fewer infections during the early phase of the outbreak?

7. In the early exponential-growth phase of the outbreak, the infection count satisfies

$$\frac{dK}{dt} = 0.15K, \quad K(0) = 50.$$

Solve this separable differential equation. According to this model, on what day will the number of cumulative infections reach 1000?

8. A more realistic model for the cumulative number of infections is given by the logistic differential equation

$$\frac{dL}{dt} = 0.15L\left(1 - \frac{L}{200,000}\right), \quad L(0) = 50,$$

where L is the number of people who have been infected after t days. Without solving the equation, describe the long-term behavior of $L(t)$. For what value of L is $\frac{dL}{dt}$ maximized? Interpret this value in the context of the outbreak.

9. A policy advisor claims: "*Strategy 2 (quarantine) has a higher initial infection rate, so it must be worse*." Using your results from Question 6, write a response evaluating this claim. Explain why comparing initial rates alone can be misleading, and why the cumulative totals given by integrals provide a better basis for comparison. Identify at least one limitation of these models that may affect the validity of the conclusions.