CHAPTER 14 PROJECT

Revenue and Cost Analysis at a Real Estate Development Firm

You are a financial planning intern at a real estate development firm. The firm is analyzing projected revenue streams from a new mixed-use building, estimating total construction costs from marginal cost rates, and comparing lease options using area-based methods. In this project, you will use indefinite integrals, Riemann sums, the Fundamental Theorem of Calculus, area between curves, and differential equations to support the firm's financial decisions.

1. The firm's marginal revenue from leasing office space is $R'(x) = 300 - 4x$ dollars per unit, where x is the number of units leased. Find the revenue function $R(x)$ by evaluating the indefinite integral of $R'(x)$. Given that revenue is \$0 when no units are leased, determine the constant of integration.

2. The marginal cost of constructing x apartment units in a new mixed-use building is given by $C'(x) = 150 + 0.6x$, where costs are measured in thousands of dollars. Find the total cost function $C(x)$ given that fixed costs are \$50,000 $\left(\text{i.e.}, C(0) = 50\right)$. What is the total cost of constructing 80 units?

3. The rate at which rental income is generated (in dollars per year) from the mixed-use building's retail floor is modeled by $I(t) = 240{,}000e^{0.05t}$, where t is the number of years since opening. Find the total rental income over the first 5 years by evaluating a definite integral. Use substitution if needed.

4. The rate of construction spending (in thousands of dollars per month) during a 12-month renovation of an office building is given by

$$F(t) = 80 + 20t - t^2, 0 \le t \le 12.$$

a. Find the total construction spending for the renovation project by evaluating a definite integral. Interpret your result.

b. Use a left-endpoint Riemann sum with $n = 4$ subintervals (each of width 3 months) to approximate the total construction spending. Compare your approximation to the exact value computed in part a.

5. The office building's value satisfies

$$\frac{dV}{dt} = 0.03V,$$

where V is value in dollars and t is measured in years. Solve this separable differential equation. If the building is worth \$5,000,000 at $t = 0$, find $V(t)$. What will the building be worth after 10 years?

6. Two different lease structures generate monthly revenue streams, modeled as follows, where t is measured in months.

Structure A: $R_A(t) = 5000 + 200t$

Structure B: $R_B(t) = 4000 + 400t$

Find the time at which the two structures generate equal monthly revenue. Then compute the area between the two curves from $t = 0$ to that intersection point and interpret the area in terms of revenue.

7. An apartment building's occupancy $G(t)$ satisfies the logistic differential equation:

$$\frac{dG}{dt} = 0.8G(1-G), G(0) = 0.1,$$

where G is the proportion of apartments that are occupied after t years. Without solving the equation, describe the long-term behavior of $G(t)$. At what occupancy level G is the rate of change $\frac{dG}{dt}$ maximized? Interpret this value in context.

8. The firm is analyzing two investment projects. The net income rates (in dollars per year) for the projects are given by $N_1'(t) = 120,000e^{-0.05t}$ and $N_2'(t) = 80,000e^{-0.02t}$, where t is in years.

a. Calculate the total income each project generates over the first 10 years.

b. Which project generates more total income over 10 years?

c. Explain why a higher initial rate does not guarantee higher long-term income.