CHAPTER 13 PROJECT

Mathematical Modeling of Cardiovascular Function and Drug Clearance

You are a biomedical engineering research assistant studying cardiovascular dynamics. In your lab, mathematical models are used to analyze blood flow rates, drug clearance from the bloodstream, changes in cardiac output, and blood vessel elasticity. In this project, you will apply derivatives of logarithmic and exponential functions, along with growth and decay models, elasticity concepts, L'Hôpital's Rule, and differentials to interpret cardiovascular data.

1. Cardiac output, measured in liters per minute (L/min), increases logarithmically during exercise according to

$$Q(t) = 2.5\ln(t+1) + 5,$$

where t is the time in minutes since exercise began. Find $Q'(t)$, which represents how quickly cardiac output is changing per minute of exercise. Note that the derivative $Q'(t)$ is measured in liters per minute per minute (L/min²). Evaluate $Q'(0)$, $Q'(4)$, and $Q'(14)$. Describe how the rate of increase in cardiac output changes throughout the exercise period.

2. A drug is cleared from the bloodstream according to the exponential decay model

$$C(t) = 80e^{-0.25t},$$

where C is the drug concentration in mg/L and t is time in hours. Find $C'(t)$. At what rate is the drug concentration changing at $t = 2$? Drug concentrations below the therapeutic threshold are ineffective. How many hours must pass before the concentration drops below the therapeutic threshold of 5 mg/L?

3. Blood flow velocity in a cylindrical artery is modeled by

$$V(r) = k\left(R^2 - r^2\right),$$

where R is the radius of the artery, r is the distance from the center, and k is a constant. Find $\frac{dV}{dr}$, the rate of change of blood flow velocity with respect to distance from the center. Determine the velocity at the center of the artery $(r = 0)$ and at the arterial wall $(r = R)$. Where within the artery is the velocity changing most rapidly?

4. In physiology, the term *elasticity* is used by analogy to elasticity of demand in economics, but it does not carry the same revenue or optimization interpretation. Here, elasticity is used as a dimensionless measure of sensitivity. Suppose the elasticity of a blood vessel wall is defined as

$$E(p) = -\frac{1}{p} \cdot \frac{V(p)}{V'(p)},$$

where p is blood pressure and $V(p)$ is vessel volume. If the vessel volume is given by $V(p) = 200p^{0.3}$, find $E(p)$. Classify the vessel expansion as elastic or inelastic, and interpret this result in a medical context.

5. Oxygen saturation (as a percentage) during recovery from exercise follows the model

$$S(t) = 100\left(1 - 0.35e^{-0.2t}\right),$$

where t is the number of minutes of rest. Find $S'(t)$. Determine the rate of oxygen recovery at $t = 5$. At what time does oxygen saturation reach 95%?

6. Identify the indeterminate form of the following limit, and then evaluate the limit using L'Hôpital's Rule.

$$\lim_{t \to 0} \frac{e^{-0.5t} - 1}{t}$$

Interpret the result as the initial rate of some biological process.

7. The function

$$Q(t) = 2.5\ln(t+1) + 5$$

models average cardiac output (in L/min) during sustained exercise. Use differentials to approximate the change in cardiac output as exercise time increases from $t = 9$ to $t = 10$ minutes. Then compute the exact change predicted by the model and compare the two results. In practice, clinicians often use mathematical models and recent trends to estimate short-term physiological changes. Suppose the model has been validated for some patient population and is used to estimate how cardiac output is expected to change over the next minute during steady exercise. How accurate is the differential approximation for this purpose?

8. Two patients are clearing the same drug but at different rates. For Patient A, the drug concentration follows the model from Question 2. The drug concentration models are as follows.

- Patient A: $C_A(t) = 80e^{-0.25t}$
- Patient B: $C_B(t) = 80e^{-0.15t}$

Find the half-life of the drug for each patient. Use differentials to estimate how much drug concentration (in mg/L) each patient loses during hour 3. Briefly explain how half-life helps clinicians anticipate changes in drug concentration over time.

9. Regarding the drug concentration model from Question 2, a medical student states: "Exponential decay means the drug will eventually reach exactly zero concentration in the bloodstream." Explain mathematically why this statement is technically false. Once the concentration $C(t)$ drops below the therapeutic threshold, the drug is considered medically ineffective regardless of the trace amount remaining. Explain why the derivative $C'(t)$ and differentials are often more useful than the function $C(t)$ itself when a doctor or pharmacist needs to determine appropriate dosing intervals for a patient.