CHAPTER 13 PROJECT

Transit Planning and Revenue Analysis

You are an analyst intern at your city's transit authority. The authority uses mathematical models to forecast ridership growth, analyze fare elasticity, estimate operating costs, and plan long-term infrastructure investments. In this project, you will apply derivatives of logarithmic and exponential functions, growth and decay models, elasticity of demand, L'Hôpital's Rule, and differentials to support data-driven planning decisions

1. Monthly ridership (in thousands of riders) is modeled by $R(t) = 250\ln(t+1) + 800$, where t is the number of months since a new route was launched. Find $R'(t)$ which represents the rate at which ridership is changing per month. Evaluate $R'(0)$ and $R'(12)$. Is ridership growing faster at launch or after one year? Explain.

2. A bus fleet depreciates in value according to $V(t) = 1{,}200{,}000e^{-0.12t}$, where t is measured in years and $V(t)$ is in dollars. Find $V'(t)$. What is the rate of depreciation at $t = 5$? After how many years will the fleet be worth less than \$400,000?

3. The transit authority models daily ridership as a function of fare price: $F(p) = 50{,}000e^{-0.04p}$, where p is the fare in dollars. Find $F'(p)$. Then calculate the elasticity of demand

$$E(p) = -\frac{pF'(p)}{F(p)}$$

and interpret your result.

4. Using the elasticity function from Question 3, evaluate $E(2.50)$ and $E(5.00)$. At each fare, classify demand as elastic, inelastic, or unit elastic. If the authority raises fares from \$2.50, will revenue increase or decrease? Explain using elasticity.

5. Daily revenue is given by $R(p) = p \cdot F(p) = 50,000pe^{-0.04p}$. Find $R'(p)$. Determine the fare that maximizes revenue. Verify that this fare corresponds to unit elasticity $(|E| = 1)$.

6. Identify the indeterminate form of the following limit and then evaluate the limit using L'Hôpital's Rule.

$$\lim_{t \to \infty} \frac{250 \ln(t+1)}{t}$$

Interpret the result in the context of long-run ridership growth per month.

7. The operating cost function is $C(x) = 800,000 + 5x + 0.0001x^2$, where x is the number of daily riders. The average cost per rider is $A(x) = \frac{C(x)}{x}$. Evaluate the limits $\lim_{x \to \infty} A(x)$ and $\lim_{x \to 0^+} A(x)$, and interpret your results.

8. Using the demand model from Question 3, $F(p) = 50,000e^{-0.04p}$, the transit authority is considering a small fare increase from \$2.50 to \$2.60. Use differentials to approximate the change in daily ridership. Then compute the exact change predicted by the model and compare the two results. How accurate is the differential approximation, and why is this type of estimate useful for short-term planning?

9. A new electric bus has a battery capacity that decays over time according to $B(t) = 100e^{-0.03t}$, where t is measured in months and $B(t)$ is the percentage of original capacity. The battery is replaced when capacity drops to 70%. How many months will pass before battery replacement is required? Use differentials to estimate the capacity loss during month 12.

10. The transit board is considering a 20% fare increase to cover a budget shortfall. Using your elasticity analysis (Questions 3 and 4), revenue model (Question 5), and differential approximation (Question 8), write a brief quantitative argument for or against the fare increase. Your argument should include the elasticity of demand at the current fare, expected changes in ridership and revenue, and at least one limitation of the mathematical models used. Based on your analysis, suggest an alternative strategy the transit authority might consider.