CHAPTER 11 PROJECT

Risk Modeling at an Insurance Company

You are an actuarial intern at a midsize insurance company. The company management uses calculus-based models to understand how policy volume, pricing, costs, staffing, advertising, and risk exposure interact. You will analyze these models using the Product Rule, the General Power Rule, implicit differentiation, analysis of increasing and decreasing intervals of a function, the First Derivative Test, and absolute extrema to support sound business decisions.

Throughout this project,

- x represents hundreds of policies;
- dollar amounts are in thousands of dollars unless stated otherwise.

1. The company models total expected claims cost as the product of the number of policies sold and the expected claim cost per policy: $C(x) = x(40 + 0.12x)$. Find $C'(x)$. Interpret $C'(x)$ in business terms.

2. The number of claims processed per day depends on the staffing level s (the number of claims adjusters): $Q(s) = \sqrt{36s + 64}$. Find $Q'(s)$. Evaluate $Q'(4)$. Explain the meaning of your result within the context of daily claims processed and staffing levels.

3. Using the expected claims cost function $C(x)$ from Question 1, determine the intervals on which $C(x)$ is increasing and decreasing. Identify all critical value(s) and classify them using the First Derivative Test. Interpret the business meaning of each result.

4. The company's total revenue is given by $R(x) = x \cdot p(x)$, where $p(x) = 120 - 0.25x$ is the average premium per policy. Write an explicit formula for $R(x)$. Then find $R'(x)$. Determine the number of policies sold that maximizes the revenue. Find the maximum revenue and interpret your result.

5. Profit is defined as revenue minus expected claims cost: $P(x)=R(x)-C(x)$ using your results from Questions 1 and 4, write an explicit formula for $P(x)$. Find $P'(x)$ and determine the number of policies sold that maximizes the profit. Compare the profit-maximizing policy volume to the revenue-maximizing volume.

6. The relationship between monthly advertising spending a (in thousands of dollars) and the number of new policy applications is modeled by $N(a)=500a^{\frac{2}{3}}$. Find $N'(a)$. Evaluate $N'(8)$ and $N'(64)$. Explain why the marginal effectiveness of advertising decreases as spending increases. Relate this behavior to the concept of *diminishing returns*.

7. The company's risk exposure depends on both policy volume x and number of claims y:

$$y^2+2xy-3x^2=600.$$

Use implicit differentiation to find $\frac{dy}{dx}$. Suppose $x=10$; find the corresponding value of y. Evaluate $\frac{dy}{dx}$ at that point. Interpret the meaning of this rate of change for the company.

8. The company models its risk-adjusted cost of setting a monthly premium p as $K(p)=0.4p^2-48p+3000$ for $50\le p\le 110$. Determine the absolute minimum risk-adjusted cost. Explain why checking endpoints is essential in business decision-making.

9. The chief actuary states, "We should always try to maximize the number of policies sold." Using your analysis of the cost, revenue, profit, and risk exposure from Questions 3, 4, 5, and 7, explain why this strategy is flawed. Discuss what happens to profitability, expected claims costs, and risk exposure as policy volume increases without bound. What quantity *should* the company seek to maximize?