C Simplifying Complex Algebraic Expressions

A **complex algebraic expression** is an expression that involves rational expressions and more than one operation. In simplifying such expressions, the rules for order of operations apply. As with complex fractions, the objective is to simplify the expression so that it is written in the form of a single reduced rational expression.

6. Simplify the following expression.

$$\frac{6}{x+4}+\frac{x}{x+4}\div\frac{x}{x-4}$$

Math Tip

Pay close attention to the order of operations when simplifying complex algebraic expressions. It may be tempting to add the numerators of the first two terms in Example 6 since they share a common denominator. However, this would give us the wrong solution since division between the second and third terms needs to happen first.

Example 6 Simplifying Complex Algebraic Expressions

Simplify the following expression.

$$\frac{4-x}{x+3}+\frac{x}{x+3}\div\frac{x}{x-3}$$

Solution

The rules for order of operations indicate that the division is to be done first, followed by the addition.

$$\begin{aligned}\frac{4-x}{x+3}+\frac{x}{x+3}\div\frac{x}{x-3}&=\frac{4-x}{x+3}+\frac{x}{x+3}\cdot\frac{x-3}{x}\\&=\frac{4-x}{x+3}+\frac{x-3}{x+3}\\&=\frac{4-x+x-3}{x+3}\\&=\frac{1}{x+3}\end{aligned}$$

Now work margin exercise 6.

Margin Exercise Answers

1. $\frac{1}{3y}$ **2.** $\frac{-6}{(x+6)^2}$ **3.** $-9xy$ **4.** $\frac{-6}{(x+6)^2}$ **5.** $-9xy$ **6.** $\frac{x+2}{x+4}$

9.5 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. The goal in simplifying complex fractions is to create a reduced _____ expression.

2. There are two methods of simplifying complex fractions. One method begins by simplifying the _____ and the _____ into single rational expressions.

3. A second method of simplifying complex fractions requires that the ___ of all denominators be found.

4. In a complex fraction, the large fraction bar is a symbol of _____.

5. An expression that involves rational expressions and more than one operation is called a/an _____ _____ expression.

6. A fraction in which the numerator and/or denominator are fractions or are the sums and/or differences of fractions is considered a/an _____ fraction.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. When simplifying complex fractions, the answer should always be reduced to lowest terms.

8. Complex fractions are those fractions in which only the denominator consists of one or more fractions itself.

9. Sometimes finding the LCM of all denominators is an important first step for simplifying complex fractions.

10. The LCM of the denominators of $\frac{2}{x-6}$ and $\frac{x}{6}$ is 6.

Practice

Simplify the following complex fractions. See Examples 1 through 5.

1. $\dfrac{\frac{2x}{3y^2}}{\frac{5x^2}{6y}}$

2. $\dfrac{\frac{6x^2}{5y}}{\frac{x}{10y^2}}$

3. $\dfrac{\frac{12x^3}{7y^2}}{\frac{3x^5}{2y}}$

4. $\dfrac{\frac{9x^2}{7y^3}}{\frac{3xy}{14}}$

5. $\dfrac{\frac{x+3}{2x}}{\frac{2x-1}{4x^2}}$

6. $\dfrac{\frac{x-2}{6x}}{\frac{x+3}{3x^2}}$

7. $\dfrac{\frac{2x-1}{x}}{\frac{2}{x}+3}$

8. $\dfrac{2-\frac{3}{x}}{\frac{x^2-4}{x}}$

9. $\dfrac{\frac{3}{x}+\frac{1}{2x}}{1+\frac{2}{x}}$

10. $\dfrac{\frac{3}{x}+\frac{5}{2x}}{\frac{1}{x}+4}$

11. $\dfrac{1+\frac{1}{x}}{1-\frac{1}{x^2}}$

12. $\dfrac{\frac{2}{y}+1}{\frac{4}{y^2}-1}$

13. $\dfrac{\frac{1}{x}+\frac{1}{3x}}{\frac{x+6}{x^2}}$

14. $\dfrac{\frac{3}{x}-\frac{6}{x^2}}{\frac{x-2}{x^2}}$

15. $\dfrac{\frac{7}{x}-\frac{14}{x^2}}{\frac{1}{x}-\frac{4}{x^3}}$

16. $\dfrac{\frac{x}{3}+\frac{1}{9x^2}}{\frac{1}{27x^2}+\frac{x}{9}}$

17. $\dfrac{\frac{1}{3}+\frac{1}{x}}{\frac{1}{2}-\frac{1}{x}}$

18. $\dfrac{\frac{x}{y}-\frac{1}{3}}{\frac{6}{y}-\frac{2}{x}}$

19. $\dfrac{\frac{2}{x}+\frac{3}{4y}}{\frac{3}{2x}-\frac{5}{3y}}$

20. $\dfrac{\frac{4}{3x}-\frac{5}{y}}{\frac{1}{3}+\frac{3}{y}}$

21. $\dfrac{1+x^{-1}}{1-x^{-2}}$

22. $\dfrac{x^{-3}+1}{1-x^{-1}}$

23. $\dfrac{1}{x^{-1}+y^{-1}}$

24. $\dfrac{x-y}{x^{-2}-y^{-2}}$

25. $\dfrac{x^{-1}+y^{-1}}{x+y}$

26. $\dfrac{y^{-2}-x^{-2}}{x+y}$

27. $\dfrac{x^{-1}+y^{-1}}{x^{-1}-y^{-1}}$

28. $\dfrac{x^{-1}+y^{-1}}{x^{-2}-y^{-2}}$

29. $\dfrac{\dfrac{4}{x}-1}{1-\dfrac{1}{x-3}}$

30. $\dfrac{x+\dfrac{3}{x-4}}{1-\dfrac{1}{x}}$

31. $\dfrac{1-\dfrac{4}{x+3}}{1-\dfrac{2}{x+1}}$

32. $\dfrac{1+\dfrac{4}{2x-3}}{1+\dfrac{x}{x+1}}$

33. $\dfrac{\dfrac{1}{x+h}-\dfrac{1}{x}}{h}$

34. $\dfrac{\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}}{h}$

35. $\dfrac{\left(2+\dfrac{1}{x+h}\right)-\left(2+\dfrac{1}{x}\right)}{h}$

36. $\dfrac{\left(\dfrac{1}{(x+h)^2}-3\right)-\left(\dfrac{1}{x^2}-3\right)}{h}$

37. $\dfrac{x^2-4y^2}{1-\dfrac{2x+y}{x-y}}$

38. $\dfrac{8x^2-2y^2}{\dfrac{4x-1}{x-y}-2}$

39. $\dfrac{\dfrac{x+1}{x-1}-\dfrac{x-1}{x+1}}{\dfrac{x+1}{x-1}+\dfrac{x-1}{x+1}}$

40. $\dfrac{\dfrac{1}{x^2-1}-\dfrac{1}{x+1}}{\dfrac{1}{x-1}+\dfrac{1}{x^2-1}}$

41. $\dfrac{\dfrac{x}{x-4}-\dfrac{1}{x-1}}{\dfrac{x}{x-1}+\dfrac{2}{x-3}}$

42. $\dfrac{\dfrac{1}{x+1}-\dfrac{x}{x+2}}{\dfrac{x}{x+2}-\dfrac{2}{x-1}}$

Simplify the following complex algebraic expressions. See Example 6.

43. $\dfrac{1}{x+1}-\dfrac{3}{2x}\cdot\dfrac{4x}{x+1}$

44. $\dfrac{4}{x}-\dfrac{2}{x^2-2x}\cdot\dfrac{x-2}{5}$

45. $\left(\dfrac{8}{x}-\dfrac{3}{4x}\right)\div\dfrac{4x+5}{x}$

46. $\left(\dfrac{2}{x}+\dfrac{5}{x-3}\right)\div\dfrac{x}{2x-6}$

47. $\dfrac{x}{x-1}-\dfrac{3}{x-1}\cdot\dfrac{x+2}{x}$

48. $\dfrac{x+3}{x+2}+\dfrac{x}{x+2}\cdot\dfrac{x-3}{x^2}$

49. $\dfrac{x-1}{x+4}+\dfrac{x-6}{x^2+3x-4}\div\dfrac{x-4}{x-1}$

50. $\dfrac{x}{x+3}-\dfrac{3}{x+5}\div\dfrac{x-2}{x^2+3x-10}$

Applications

Solve.

51. To calculate the average rate of a two-part commute, where each part is the same distance, the following formula is used.

$$\frac{2d}{\frac{d}{r_1}+\frac{d}{r_2}}$$

In the formula, d is the commute distance traveled one way, r_1 is the rate, or speed, during the first part of the trip, and r_2 is the rate during the second part of the trip.

a. Simplify the expression.

b. Calculate the average rate of the trip if you can travel 35 miles per hour during the first part of the trip and 60 miles per hour during the second part of the trip. Round your answer to the nearest tenth.

c. Use the answer from part b. and the formula $d = rt$ to calculate how long the commute took if the total distance of the trip was 80 miles. Round your answer to the nearest tenth.

52. The average percent yield (APY) of an annuity is the annual interest rate earned in a given year that accounts for the effects of compounding. The APY acts as the interest rate for a simple interest account and is larger than the stated interest rate on the compound interest account. The formula to calculate the APY on an annuity after 2 years is

$$\text{APY} = \left(1+\frac{r}{2}\right)^2 - 1,$$

where r is the stated interest rate.

a. Simplify the expression for APY and write as a single rational expression.

b. Using the original formula, calculate the APY for an annuity whose interest rate is 6%. Do not round.

c. Using the expression in part a., calculate the APY for an annuity whose interest rate is 6%. Do not round.

d. Does the result from part c. match the result from part b.? Explain why or why not.

e. How much larger is the APY than the interest rate?

f. Why do you think the APY is larger than the interest rate? Write a complete sentence.

Writing & Thinking

53. Some complex fractions involve the sum (or difference) of complex fractions. Beginning with the outermost denominator, simplify each of the following expressions.

a. $1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+1}}}$ **b.** $2-\dfrac{1}{2-\dfrac{1}{2-\dfrac{1}{2-1}}}$ **c.** $x+\dfrac{1}{x+\dfrac{1}{x+\dfrac{1}{x+1}}}$