### Solution

**a.** $x^3 - 8 = x^3 - 2^3$

$$= (x-2)(x^2 + 2\cdot x + 2^2)$$

$$= (x-2)(x^2 + 2x + 4)$$

**Note:** Remember that the second polynomial is not a perfect square trinomial and cannot be factored.

**b.** $x^6 + 64y^3 = (x^2)^3 + (4y)^3$

$$= (x^2 + 4y)\left[(x^2)^2 - 4y\cdot x^2 + (4y)^2\right]$$

$$= (x^2 + 4y)(x^4 - 4x^2y + 16y^2)$$

**c.** Factor out the GCF first. Then factor the **difference of two cubes**.

$$16y^{12} - 250 = 2(8y^{12} - 125)$$

$$= 2\left[(2y^4)^3 - 5^3\right]$$

$$= 2(2y^4 - 5)\left[(2y^4)^2 + (5)(2y^4) + 5^2\right]$$

$$= 2(2y^4 - 5)(4y^8 + 10y^4 + 25)$$

***Now work margin exercise 4.***

***Margin Exercise Answers***

**1. a.** $7a(x-7)(x+7)$ **b.** $(y^3+10)(y^3-10)$ **2. a.** Not factorable **b.** $5(9x^2+4)$
**3. a.** $(z+20)^2$ **b.** $(y-7)^2$ **c.** $3z(x-3y)^2$ **d.** $(y+4-z)(y+4+z)$
**4. a.** $(y-3)(y^2+3y+9)$ **b.** $(2y-x^2)(4y^2+2x^2y+x^4)$ **c.** $6(2x^4-5)(4x^8+10x^4+25)$

# 8.5 Exercises

## Concept Check

**Fill-in-the-Blank.** Complete each sentence using information found in this section.

1. Factoring a perfect square trinomial gives a square ___________.
2. In a perfect square trinomial, both the first and last terms must be perfect _______.
3. If the first term of a perfect square trinomial is $x^2$, and the last term is of the form $a^2$, then the middle term must be of the form ______ or ______.
4. The formula for factoring the difference of cubes is $x^3 - a^3 =$ ___________.
5. The formula for factoring the sum of two cubes is $x^3 + a^3 =$ ___________.
6. The first 6 perfect cubes are 1, 8, ___, ___, ___, and ___.

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The expression $x^2 + 20x + 100$ is a perfect square trinomial.
8. When factoring polynomials, always look for a common monomial factor first.
9. The sum of two squares, $(x^2 + a^2)$, is factorable.
10. Sixty-four is a perfect square and a perfect cube.

## Practice

Completely factor each of the given polynomials. If a polynomial cannot be factored, write "not factorable." See Examples 1 through 4.

1. $x^2 - 25$
2. $y^2 - 121$
3. $81 - y^2$
4. $25 - z^2$
5. $2x^2 - 128$
6. $3x^2 - 147$
7. $4x^4 - 64$
8. $5x^4 - 125$
9. $y^2 + 100$
10. $4x^2 + 49$
11. $y^2 - 16y + 64$
12. $z^2 + 18z + 81$
13. $-4x^2 + 100$
14. $-12x^4 + 3$
15. $9x^2 - 25$
16. $4x^2 - 49$
17. $y^2 - 10y + 25$
18. $x^2 + 12x + 36$
19. $4x^2 - 4x + 1$
20. $49x^2 - 14x + 1$
21. $25x^2 + 30x + 9$
22. $9y^2 + 12y + 4$
23. $16x^2 - 40x + 25$
24. $9x^2 - 12x + 4$
25. $4x^3 - 64x$
26. $50x^3 - 8x$
27. $2x^3y + 32x^2y + 128xy$
28. $3x^2y - 30xy + 75y$
29. $y^2 + 6y + 9$
30. $y^2 + 4y + 4$
31. $x^2 - 20x + 100$
32. $25x^2 - 10x + 1$
33. $x^4 + 10x^2y + 25y^2$
34. $16x^4 + 8x^2y + y^2$
35. $x^3 - 125$
36. $x^3 - 64$
37. $y^3 + 216$
38. $y^3 + 1$
39. $x^3 + 27y^3$
40. $8x^3 + 1$
41. $x^2 + 64y^2$
42. $3x^3 + 81$
43. $4x^3 - 32$
44. $64x^3 + 27y^3$
45. $54x^3 - 2y^3$
46. $3x^4 + 375xy^3$
47. $x^3y + y^4$
48. $x^4y^3 - x$
49. $x^2y^2 - x^2y^5$
50. $2x^2 - 16x^2y^3$
51. $24x^4y + 81xy^4$
52. $x^6 - 64y^3$
53. $x^6 - y^9$
54. $64x^2 + 1$
55. $27x^3 + y^6$
56. $x^3 + 64z^3$
57. $8x^3 + y^3$
58. $x^3 + 125y^3$
59. $8y^3 - 8$
60. $36x^3 + 36$
61. $9x^2 - y^2$
62. $x^2 - 4y^2$
63. $x^4 - 16y^4$

**64.** $81x^4 - 1$

**65.** $(x-y)^2 - 81$

**66.** $(x+2y)^2 - 25$

**67.** $(x^2 - 2xy + y^2) - 36$

**68.** $(x^2 + 4xy + 4y^2) - 25$

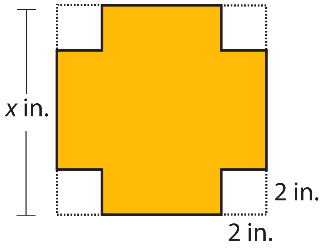
**69.** $(16x^2 + 8x + 1) - y^2$

**70.** $x^2 - (y^2 + 6y + 9)$

Solve.

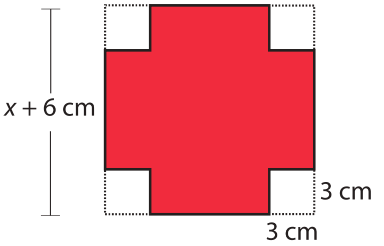
**71.** **a.** Represent the area of the shaded region of the square shown below as the difference of two squares.

**b.** Use the factors of the expression in part a. to draw (and label the sides of) a rectangle that has the same area as the shaded region.




**72.** **a.** Use a polynomial function to represent the area of the shaded region of the square.

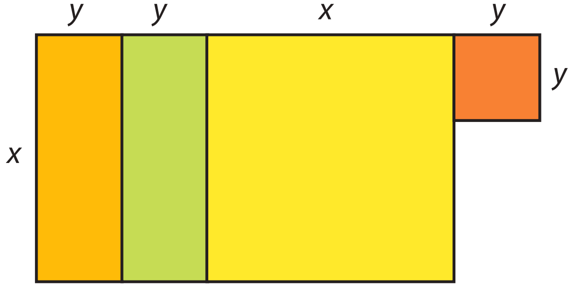
**b.** Use a polynomial function to represent the perimeter of the shaded figure.




## Writing & Thinking

**73.** **a.** Show that the sum of the areas of the rectangles and squares in the figure is a perfect square trinomial.

**b.** Rearrange the rectangles and squares in the form of a square and represent its area as the square of a binomial.



**74.** Compound interest is interest earned on interest. If a principal $P$ is invested and compounded annually (once a year) at a rate of $r$, then the amount, $A_1$ accumulated in one year is $A_1 = P + Pr$.
In factored form, we have $A_1 = P + Pr = P(1+r)$.
At the end of the second year the amount accumulated is $A_2 = (P+Pr)+(P+Pr)r$.

**a.** Write the expression for $A_2$ in factored form similar to that for $A_1$.

**b.** Write an expression for the amount accumulated in three years, $A_3$, in factored form.

**c.** Write an expression for $A_n$ the amount accumulated in $n$ years.

**d.** Use the formula you developed in part c. and your calculator to find the amount accumulated if $10,000 is invested at 6% and compounded annually for 20 years.

**75.** You may have heard of (or studied) the following rules for division of an integer by 3 and 9:

**1.** An integer is divisible by 3 if the sum of its digits is divisible by 3.

**2.** An integer is divisible by 9 if the sum of its digits is divisible by 9.

The proofs of both **1.** and **2.** can be started as follows.

Let *abc* represent a three-digit integer.

$$\begin{aligned} \text{Then } abc &= 100a + 10b + c \\ &= (99+1)a + (9+1)b + c \\ &= (\text{now you finish the proofs}) \end{aligned}$$

Use the pattern just shown and prove both **1.** and **2.** for a four-digit integer.