Solution

a. To begin, factor out the common monomial $2x$.

$$6x^3 - 8x^2 + 2x = 2x(3x^2 - 4x + 1)$$

Now, factor the trinomial $3x^2 - 4x + 1$. Since the middle term is $-4x$ and the constant is $+1$, we know that the two factors of 1 must both be negative, -1 and -1. We also know that $\mathbf{F} = 3x^2 = 3x \cdot x$. This gives

$$3x^2 - 4x + 1 = (3x - 1)(x - 1). \qquad -3x - x = -4x$$

$-x$, $-3x$

Thus, $6x^3 - 8x^2 + 2x = 2x(3x^2 - 4x + 1) = 2x(3x - 1)(x - 1)$. Be careful to include the monomial term in the answer.

Check

Check the factorization by multiplying.

$$\begin{aligned} 2x(3x-1)(x-1) &= 2x(3x^2 - 3x - x + 1) \\ &= 2x(3x^2 - 4x + 1) \\ &= 6x^3 - 8x^2 + 2x \end{aligned}$$

The original polynomial

b. $$\begin{aligned} -2x^2 - x + 6 &= -1(2x^2 + x - 6) \\ &= -1(2x - 3)(x + 2) \end{aligned}$$

Notice that factoring out -1 gives a positive leading coefficient for the trinomial.

c. $10x^3 + 5x^2 + 5x = 5x(2x^2 + x + 1)$

Now consider the trinomial: $2x^2 + x + 1 = (2x + ?)(x + ?)$.

The factors of $+1$ need to be $+1$ and $+1$, but $(2x + 1)(x + 1) = 2x^2 + 3x + 1$.

So there is no way to factor and get a middle term of $+x$ for the product. This trinomial, $2x^2 + x + 1$, is **not factorable**.

We have

$10x^3 + 5x^2 + 5x = 5x(2x^2 + x + 1)$. Factored completely

Now work margin exercise 2.

Margin Exercise Answers

1. a. $(x + 6)(x + 2)$ **b.** $(4u - 7)(2u + 3)$ **2. a.** $4x(2x - 1)(x - 1)$ **b.** $7x(3x^2 + 7x - 1)$

Note

Remember, *to factor completely* means to find factors of the polynomial, none of which are themselves factorable. Thus,

$2x^2 + 12x + 10 = (2x + 10)(x + 1)$

is not factored completely because $2x + 10 = 2(x + 5)$.

We could write

$$\begin{aligned} 2x^2 + 12x + 10 &= (2x + 10)(x + 1) \\ &= 2(x + 5)(x + 1). \end{aligned}$$

This problem can be avoided by first factoring out the GCF (in this case, 2).

8.3 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. When using the trial-and-error method to factor a trinomial of the form $ax^2 + bx + c$, you first need to list all possible combinations of _____ of a and c, in their respective "First" and "Last" positions, according to the FOIL method.

2. The second step is to check the sums of the _____ in the O and I positions in the list until you find the sum to be c.

3. If none of these sums is c, the trinomial is not _____.

4. Look at the _____ term to determine what signs to use for the constants in the factors.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

5. The trial-and-error method of factoring a trinomial follows the same steps as the FOIL method of multiplication.

6. Factoring can be checked by multiplying the factors and verifying that the product matches the original polynomial.

Practice

Completely factor each polynomial. If a polynomial cannot be factored, write "not factorable."

1. x^2+5x+6
2. x^2-6x+8
3. $2x^2-3x-5$
4. $3x^2-4x-7$
5. $6x^2+11x+5$
6. $4x^2-11x+6$
7. $-x^2+3x-2$
8. $-x^2-5x-6$
9. $x^2-3x-10$
10. $x^2-11x+10$
11. $-x^2+13x+14$
12. $-x^2+12x-36$
13. $x^2+8x+64$
14. x^2+2x+3
15. $-2x^3+x^2+x$
16. $-2y^3-3y^2-y$
17. $4t^2-3t-1$
18. $2x^2-3x-2$
19. $5a^2-a-6$
20. $3a^2+4a+1$
21. $7x^2+5x-2$
22. $4x^2+23x+15$
23. $8x^2-10x-3$
24. $6x^2+23x+21$
25. $9x^2-3x-20$
26. $4x^2+40x+25$
27. $12x^2-38x+20$
28. $12b^2-12b+3$
29. $3x^2-7x+2$
30. $7x^2-11x-6$
31. $9x^2-6x+1$
32. $4x^2+4x+1$
33. $6y^2+7y+2$
34. $12y^2-7y-12$
35. $x^2-46x+45$
36. $x^2+6x-16$
37. $3x^2+9x+5$
38. $5a^2-7a+2$
39. $12m^2n-45mn-12n$
40. $30a^3b-28a^2b+6ab$
41. x^2+x+1
42. x^2+2x+2

Writing & Thinking

43. It is true that $2x^2+10x+12=(2x+6)(x+2)=(2x+4)(x+3)$. Explain how the trinomial can be factored in two ways. Is there some kind of error?

44. It is true that $5x^2-5x-30=(5x-15)(x+2)$. Explain why this is not the completely factored form of the trinomial.