Completion Example Answers

6. $x = 3 - y$; $3 - y$;

$$\begin{aligned} 2(3-y)-y &= 12 \\ 6-2y-y &= 12 \\ 6-3y &= 12 \\ -3y &= 6 \\ y &= -2 \\ x &= 3-(-2) = 5 \end{aligned}$$

The solution to the system is $(5, -2)$.

Margin Exercise Answers

1. $(3, -5)$ **2.** $(10, 1)$ **3.** No solution **4.** $(x, 1+2x)$ or $\left(\frac{y-1}{2}, y\right)$ **5.** $(-3, 9)$ **6.** $(-1, 3)$

6.2 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. The first step when solving a system of equations using the method of substitution is to solve for one of the _____ in one of the equations.

2. The second step is to substitute the resulting expression into the _____ expression.

3. If the equation formed after substitution is never true, then the system has _____ solution(s).

4. If the equation formed after substitution is always true, then the system has a/an _____ number of solutions.

5. After solving the equation formed after substitution, the value of the variable is substituted into one of the original expressions to find the value of the other variable. This is known as _____ _____.

6. The solution should be checked in _____ equations.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The method of substitution reduces the problem from one of solving two equations in two variables to solving one equation in one variable.

8. The method of substitution is most often used when one of the equations is impossible to graph.

9. The method of substitution is more accurate than the graphing method.

10. When using the method of substitution, you should always solve the first equation for x.

Practice

Use the method of substitution to solve each system. See Examples 1 through 6.

1. $\begin{cases} x+y=6 \\ y=2x \end{cases}$

2. $\begin{cases} 5x+2y=21 \\ x=y \end{cases}$

3. $\begin{cases} 3x-7=y \\ 2y=6x-14 \end{cases}$

4. $\begin{cases} y=3x+4 \\ 2y=3x+5 \end{cases}$

5. $\begin{cases} x=3y \\ 3y-2x=6 \end{cases}$

6. $\begin{cases} 4x=y \\ 4x-y=7 \end{cases}$

7. $\begin{cases} x-5y+1=0 \\ x=7-3y \end{cases}$

8. $\begin{cases} 2x+5y=15 \\ x=y-3 \end{cases}$

9. $\begin{cases} 7x+y=9 \\ y=4-7x \end{cases}$

10. $\begin{cases} 3y+5x=5 \\ y=3-2x \end{cases}$

11. $\begin{cases} 3x-y=7 \\ x+y=5 \end{cases}$

12. $\begin{cases} 4x-2y=5 \\ y=2x+3 \end{cases}$

13. $\begin{cases} 3x+5y=-13 \\ y=3-2x \end{cases}$

14. $\begin{cases} 15x+5y=20 \\ y=-3x+4 \end{cases}$

15. $\begin{cases} x-y=5 \\ 2x+3y=0 \end{cases}$

16. $\begin{cases} 4x=8 \\ 3x+y=8 \end{cases}$

17. $\begin{cases} 2y=5 \\ 3x-4y=-4 \end{cases}$

18. $\begin{cases} x+y=8 \\ 3x+2y=8 \end{cases}$

19. $\begin{cases} y=2x-5 \\ 2x+y=-3 \end{cases}$

20. $\begin{cases} 2x+3y=5 \\ x-6y=0 \end{cases}$

21. $\begin{cases} x+5y=1 \\ x-3y=5 \end{cases}$

22. $\begin{cases} 3x+8y=-2 \\ x+2y=-1 \end{cases}$

23. $\begin{cases} 9x+3y=6 \\ 3x=2-y \end{cases}$

24. $\begin{cases} 5x+2y=-10 \\ 10x=-3-4y \end{cases}$

25. $\begin{cases} x-2y=-4 \\ 3x+y=-5 \end{cases}$

26. $\begin{cases} x+4y=3 \\ 3x-4y=7 \end{cases}$

27. $\begin{cases} 3x-y=-1 \\ 7x-4y=0 \end{cases}$

28. $\begin{cases} x+5y=-1 \\ 2x+7y=1 \end{cases}$

29. $\begin{cases} x+3y=5 \\ 3x+2y=7 \end{cases}$

30. $\begin{cases} 3x-4y-39=0 \\ 2x-y-13=0 \end{cases}$

31. $\begin{cases} \frac{1}{4}x-\frac{3}{2}y=-5 \\ -x+6y=20 \end{cases}$

32. $\begin{cases} \frac{-4}{3}x+2y=7 \\ \frac{8}{3}x-4y=-5 \end{cases}$

33. $\begin{cases} 6x-y=15 \\ 0.2x+0.5y=2.1 \end{cases}$

34. $\begin{cases} x+2y=3 \\ 0.4x+y=0.6 \end{cases}$

35. $\begin{cases} 0.2x-0.1y=0 \\ y=x+10 \end{cases}$

36. $\begin{cases} 0.1x-0.2y=1.4 \\ 3x+y=14 \end{cases}$

37. $\begin{cases} 3x-2y=5 \\ y=1.5x+2 \end{cases}$

38. $\begin{cases} x=2y-7.5 \\ 2x+4y=-15 \end{cases}$

39. $\begin{cases} \frac{1}{2}x+\frac{1}{3}y=4 \\ 3x+2y=24 \end{cases}$

40. $\begin{cases} \frac{1}{3}x+\frac{1}{7}y=2 \\ 7x+3y=42 \end{cases}$

41. $\begin{cases} \frac{x}{3}+\frac{y}{5}=1 \\ x+6y=12 \end{cases}$

42. $\begin{cases} \frac{x}{5}+\frac{y}{4}-3=0 \\ \frac{x}{10}-\frac{y}{2}+1=0 \end{cases}$

Applications

Each of the following applications has been modeled using a system of equations. Use the method of substitution to solve each system. (**Note:** Some of these exercises were also given in Section 6.1. Check to see that you arrived at the same answers by both methods.)

43. The sum of two numbers is 25 and their difference is 15. What are the two numbers?
Let x be one number
and y be the other number.

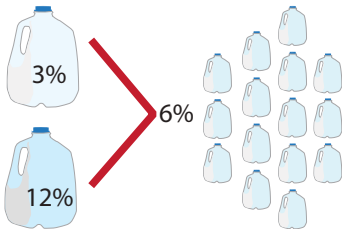
The corresponding modeling system is $\begin{cases} x+y=25 \\ x-y=15 \end{cases}$

44. The perimeter of a rectangle is 50 meters and the length is 5 meters longer than the width. Find the dimensions of the rectangle.
Let x be the length and y be the width.

The corresponding modeling system is $\begin{cases} 2x+2y=50 \\ x-y=5 \end{cases}$

45. OSHA recommends that swimming pool owners clean their pool decks with a solvent composed of a 12% chlorine solution and a 3% chlorine solution. Fifteen gallons of the solvent consists of 6% chlorine. How much of each of the mixing solutions were used?
Let x be the number of gallons of the 12% solution
and y be the number of gallons of the 3% solution.
The corresponding modeling system is

$$\begin{cases} x+y=15 \\ 0.12x+0.03y=0.06(15) \end{cases}$$



46. A student bought a calculator and a textbook for a course in algebra. He told his friend that the total cost was \$170 (without tax) and that the calculator cost \$20 more than twice the cost of the textbook. What was the cost of each item?
Let x be the cost of the calculator
and y be the cost of the textbook.
The corresponding modeling system is

$$\begin{cases} x+y=170 \\ x=2y+20 \end{cases}$$

47. A fitness center manager is trying to decide whether to charge an enrollment fee of \$25 with a monthly rate of \$50 or an enrollment fee of \$100 with a monthly rate of \$25. After how many months would it be more profitable for the manager to choose the lower enrollment fee and the higher monthly rate? Round up to the nearest month.

The corresponding modeling system is $\begin{cases} y=50x+25 \\ y=25x+100 \end{cases}$

48. Connor is retiling the backsplash of his kitchen counter. He plans on using square tiles that measure 2 inches by 2 inches and rectangular tiles that measure 2 inches by 4 inches. The backsplash measures 6 inches by 60 inches. He wants to use an equal number of rectangular tiles and square tiles. How many of each tile will he need to buy?

a. Find the area of each size tile and the area of the backsplash.

b. Write two equations to represent the situation. Use the variable s to represent the number of square tiles and the variable r to represent the number of rectangular tiles.

c. Solve the system of equations by substitution.

d. What does the solution mean? Write a complete sentence.

e. If the square tiles come in boxes of 30 and the rectangular tiles come in boxes of 18, how many boxes of each type of tile will Connor need to buy to retile the backsplash?

49. Harper is buying a gift for her friend's wedding. She found two gifts at two different stores to choose between. She plans on personalizing the selected gift with an engraved message. The first gift costs \$45 and the store charges \$0.15 per letter engraved. The second gift costs \$55 and the store charges \$0.10 per letter engraved. How many letters would the message need to contain for the two gifts to be the same cost?

a. Write two equations to represent the situation. Use the variable c to represent the total cost of the gift with engraving and the variable t to represent the number of letters in the engraved message.

b. Solve the system of equations by substitution.

c. What does the solution mean? Write a complete sentence.

d. How many letters must the engraved message contain for the second gift to be the less expensive option?

50. Carlos is buying supplies for the tutoring center where he works. He was given a budget of \$230 to spend. Calculators cost \$15 each and packs of paper are \$2.50 each. He would like to buy twice as many packs of paper as calculators. How many calculators and notebooks can Carlos buy and stay in budget? (**Note:** The tutoring center is non-profit organization and therefore does not have to pay sales tax.)

a. Write two equations to represent the situation. Use the variable c to represent the number of calculators and the variable p to represent the number of packs of paper.

b. Solve the system of equations by substitution. Write the solutions in decimal form.

c. What does the solution mean? Write a complete sentence.

d. Does the answer to part c. make sense? Explain why.

e. Based on your answer from part d., how many calculators should Carlos buy to stay within budget and to buy twice as many packs of paper as calculators?

f. If the tutoring center has at most 12 students at a time, will each student have a new calculator to work with?

Writing & Thinking

51. Explain the advantages of solving a system of linear equations

a. by graphing,

b. by substitution.