Solution

There are no values of x for which $|x| = -3$. The absolute value can never be negative. There is **no solution**.

Now work margin exercise 10.

Example 11 Application: Solving Absolute Value Equations

During the manufacturing of machine parts, certain measurements of the part must stay within a tolerance range or the part will be defective. The quality control manager determines that the maximum amount the length of a screw can vary from its targeted length of 12 millimeters and not be defective is 2 millimeters. If x represents the maximum acceptable difference in length from the target, either positive or negative, then $|x| = 2$. What are the maximum amounts that the screw length can vary from its target?

11. A quality control manager determines that the maximum amount the width of a latch can vary from its targeted length and not be defective is 3 millimeters. If y represents the maximum acceptable error, either positive or negative, then what are the maximum amounts that the hatch width can vary from its target?

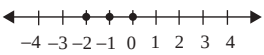
Solution

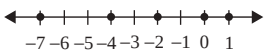
To find the maximum the screw length can vary from its target, solve the equation $|x| = 2$ millimeters. This gives the following two values for x: $x = -2$ millimeters and $x = 2$ millimeters.

Thus, the maximum amounts that the screw length can vary are –2 millimeters and 2 millimeters.

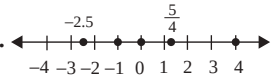
Now work margin exercise 11.

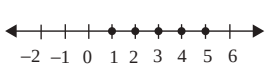
Margin Exercise Answers

1. a. –10 **b.** +8 **2. a.** 

b. 

3. a. 1, 20 **b.** –6, 1, 20 **c.** $-6, -\frac{1}{7}, 1, 20$ **d.** All numbers in S are real numbers.

4. 

5. 

6. a. True **b.** False **c.** True **d.** True **e.** False **7. a.** 4 **b.** 3.3 **c.** –7.4 **8.** True **9.** $z = 3, -3$ **10.** No solution **11.** −3 millimeters and 3 millimeters

3.1 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. The set of numbers that includes the whole numbers and their opposites is the set of _____
2. A number that can be written as a fraction is a/an _____ number.
3. Infinite nonrepeating decimal numbers are _____ numbers.
4. A number's distance from 0 on a number line is the number's _____.

5. The _____ of a number is the point that corresponds to the number on a number line.

6. The symbols < and > are known as _____ symbols.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. On a number line, smaller numbers are always to the left of larger numbers.

8. The absolute value of a negative number is a positive number.

9. All whole numbers are also integers.

10. Zero is a positive number.

Practice

Find the opposite of each integer. See Example 1.

1. -3

2. -7

3. 0

4. -0

5. $+2$

6. $+6$

Graph each set of real numbers on a real number line. See Examples 2, 4, and 5.

7. $\{1, 2, 5, 6\}$

8. $\{-3, -2, 0, 1\}$

9. $\{2, -3, 0, -1\}$

10. $\{-2, -1, 4, -3\}$

11. $\left\{0, -1, \frac{7}{4}, 3, 1\right\}$

12. $\left\{-2, -1, -\frac{1}{3}, 2\right\}$

13. $\left\{-\frac{3}{4}, 0, 2, 3.6\right\}$

14. $\left\{-3.4, -2, -0.5, 1, \frac{5}{2}\right\}$

15. $\left\{-\frac{7}{2}, -1.5, 1, \frac{4}{3}, 2\right\}$

16. $\left\{-4, -\frac{7}{3}, -1, 0.2, \frac{5}{2}\right\}$

17. all whole numbers less than 4

18. all negative integers greater than -4

19. all whole numbers less than 0

20. all natural numbers less than or equal to -1

List the numbers in the set $A = \left\{-7, -\sqrt{6}, -2, -\frac{5}{3}, -1.4, 0, \frac{3}{5}, \sqrt{5}, \sqrt{11}, 4, 5.9, 8\right\}$ that are described in each exercise. See Example 3.

21. Natural numbers

22. Whole numbers

23. Integers

24. Irrational numbers

25. Rational numbers

26. Real numbers

Fill in each blank with the appropriate symbol that will make the statement true: <, >, or =. See Examples 6 through 8.

27. $4 \_\_\_ 6$

28. $-3 \_\_\_ 1$

29. $-2 \_\_\_ -4$

30. $-8 \_\_\_ 0$

31. $-20 \_\_\_ -19$

32. $-67 \_\_\_ -50$

33. $-(-4.3) \_\_\_ 4.3$

34. $5.6 \_\_\_ -(-8.7)$

35. $-\frac{3}{4} \_\_\_ -1$

36. $-2.3 \_\_\_ -2\frac{3}{10}$

37. $\frac{1}{3} \_\_\_ \frac{1}{2}$

38. $-\frac{1}{2} \_\_\_ -\frac{1}{3}$

39. $|-4| \_\_\_ 4$

40. $|7| \_\_\_ -7$

41. $|-8| \_\_\_ -8$

42. $-15 \_\_\_ |-15|$

Determine whether each statement is true or false. If a statement is false, rewrite it in a form that is a true statement. (There may be more than one way to correct a statement.) See Examples 6 and 8.

43. $0 = -0$

44. $-22 < -16$

45. $-9 > -8.5$

46. $-17 \le 17$

47. $4.7 \ge 3.5$

48. $|-5| = 5$

49. $-|-7| \ge -|7|$

50. $|-8| \ge 4$

51. $-|-3| < -|4|$

52. $\left|-\frac{5}{2}\right| < 2$

List the possible values for x for each statement. See Examples 9 and 10.

53. $|x| = 5$

54. $|x| = 8$

55. $|x| = 2$

56. $|x| = 0$

57. $|x| = -6$

58. $|x| = -1$

59. $|x| = 23$

60. $|x| = 105$

Choose the response that correctly completes each sentence. Assume that the variables represent integers. In each problem, give two examples that illustrate your reasoning.

61. $|a|$ is (never, sometimes, always) equal to a.

62. $|x|$ is (never, sometimes, always) equal to $-x$.

63. $|y|$ is (never, sometimes, always) equal to a positive integer.

64. $|x|$ is (never, sometimes, always) greater than x.

Applications

Solve. Represent each quantity with a signed integer.

65. The Alvin is a manned deep-ocean research submersible that has explored the wreck of the Titanic. The operating depth of the Alvin is 4500 meters below sea level.

66. The Mariana trench is the deepest known location on the Earth's ocean floor. The deepest known part of the Mariana Trench is approximately 11 kilometers below sea level.

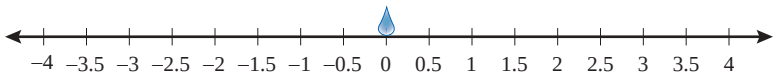
67. Mount Everest is considered to be the highest mountain on Earth. Its peak reaches to a height of approximately 8844 meters.

68. The lowest temperature ever recorded was at the Vostok Station on Antarctica. On July 21, 1983, the temperature was approximately 128 degrees Fahrenheit below zero.

69. Terrence placed a drop of colored water on the center of a white strand of yarn and measured how much the color spread. Before placing the drop, he predicts that the color will spread no more than 3 inches away from the initial drop.

a. Write an absolute value inequality using the variable x to represent the predicted spread.

b. Graph the solution set of integers for the absolute value inequality from part a. on the given number line, placing the initial drop at the point 0.

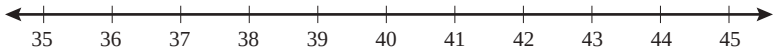


70. A ready-to-assemble bookcase contains wooden boards that have predrilled holes along with the screws and washers needed for assembly. The screws used to assemble the bookcase need to have a length of 38 mm with a tolerance of 2 mm. If the screw is too short, it won't be able to hold the pieces of wood together. If the screw is too long, it might stick out of the other end of the board.

a. What is the largest length the screws can have before they are too long?

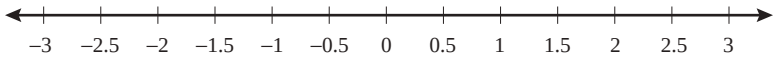
b. What is the smallest length the screws can have before they are too short?

c. Graph the tolerance of the screw. (Graph only the integers in the tolerance range.)



71. A freezer in a biology lab is supposed to be kept at 0 °C. A lab assistant places a thermometer in the freezer and marks down the temperature every half hour. She records the following temperatures in degrees Celsius: $\left\{2, -\frac{1}{2}, \frac{1}{2}, 1\frac{1}{2}, -\frac{5}{2}\right\}$.

a. Graph the set of temperatures on a number line.



b. Which value is the furthest away from 0?

72. During the manufacture of machine parts, certain measurements of the part must stay within a tolerance range or the part will be defective. The quality control manager determines that the maximum amount the length of a screw can vary from its targeted length of 15 millimeters and not be defective is 3 millimeters. If x represents the maximum acceptable difference in length from the target, either positive or negative, then $|x| = 3$.

a. Solve the equation $|x| = 3$ to determine the maximum amounts that the screw length can vary from its target length.

b. What do the answers from part a. mean?

c. Determine the maximum and minimum lengths of a screw with the given tolerance range.

73. A carpenter needs to cut a board 1 meter long to create the backboard for a bookshelf. He has determined that the maximum amount the length of the board can vary from its required length and not cause structural issues is 5 mm. If x represents the maximum acceptable difference in length from the target, either positive or negative, then $|x| = 5$.

a. Solve the equation $|x| = 5$ to determine the maximum amounts that the board length can vary from its target length.

b. What do the answers from part a. mean?

c. Determine the maximum and minimum lengths of a board with the given tolerance range.

74. Cynthia believes that the ideal daytime temperature for growing tomato plants is 77 °F, and the plants will be okay as long as the temperature does not vary more than 7 °F from that ideal temperature during the day.

a. How low can the daytime temperature be to accommodate the tomato plant growth?

b. How high can the daytime temperature be to accommodate the tomato plant growth?

75. The following table shows the elevation of six California cities. (Sea level is defined to have 0 feet of elevation.)

City	Elevation (in feet)
Alameda	50
Death Valley	−282
El Centro	−39
Fresno	296
Salton City	−125
Windsor	118

a. Which of these cities has an elevation farthest away from sea level?

b. Which of these cities has an elevation closest to sea level?

Writing & Thinking

76. Give one example each of the use of a positive number, a negative number, and the number zero (outside of a class).

77. Explain, in your own words, how an expression such as $-y$ might represent a positive number.

78. Compare and contrast absolute value with opposites.