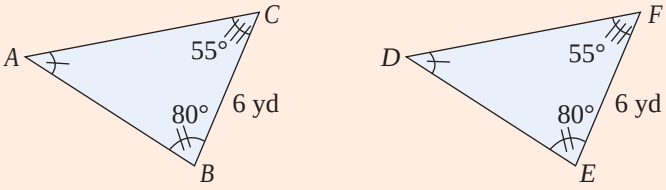
Determining Congruent Triangles (cont.)

3. Angle-Side-Angle (ASA)

If two triangles are such that two angles in one triangle are congruent to two angles in the other triangle and the lengths of the sides included between the angles are equal, then the two triangles are congruent.

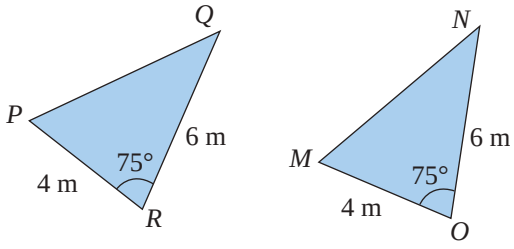


Triangle *ABC* is congruent to triangle *DEF* by ASA.

DEFINITION

Example 7 Determining Whether Triangles are Congruent

Determine whether triangles *PQR* and *MNO* are congruent.

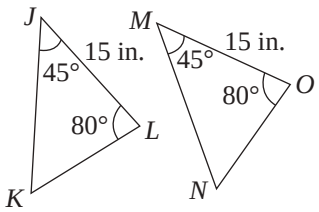


Solution

From the figure we see that $PR = MO = 4$, $QR = NO = 6$, and $m\angle R = m\angle O = 75$. As $\angle R$ and $\angle O$ are included between the pairs of equal-length sides, the two triangles are congruent by SAS (Side-Angle-Side).

Now work margin exercise 7.

7. Determine whether triangles *JKL* and *MNO* are congruent. If they are congruent, state the property that confirms that they are congruent.



Margin Exercise Answers

1. a. Right **b.** Obtuse **c.** Straight **2. a.** $\angle DOC \cong \angle FOA$, $\angle AOB \cong \angle DOE$, and $\angle FOE \cong \angle BOC$ **b.** $\angle DOC = 40°$ **c.** $m\angle FOE = 50°$ **d.** Three angles angle adjacent to $\angle DOC$ are $\angle BOC$, $\angle DOE$, and $\angle DOF$. **3.** Scalene **4.** Yes **5.** b **6.** 2.5 cm **7.** Yes, by ASA

2.2 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. Two rays with a common endpoint, called a vertex, form a/an _____.

2. An angle with a measure less than 90° is a/an _____ angle.

3. A triangle with no equal sides is a/an _____ triangle.

4. When the measures of the three angles in a triangle are added, the sum is _____ degrees.

5. Similar triangles have the same _____ even though they may not have the same size.

6. In congruent triangles, the lengths of corresponding sides are _____.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note**: There may be more than one acceptable change.)

7. The sum of the measures of two complementary angles is equal to the measure of one right angle.

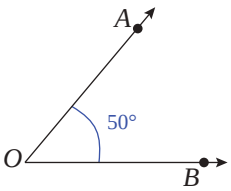
8. Adjacent angles are two angles that share a vertex and a common side but do not overlap.

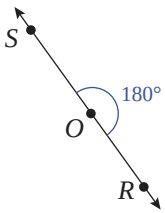
9. A triangle with three angles that each measure less than 90 degrees is an obtuse triangle.

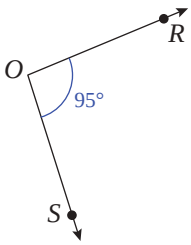
10. Congruent triangles have corresponding angles that are equal.

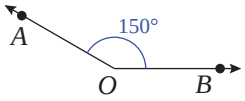
Practice

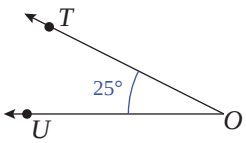
Classify each angle as acute, right, obtuse, or straight.

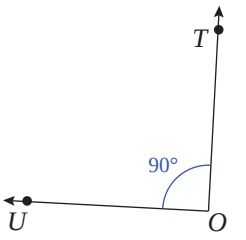
1. 

2. 

3. 

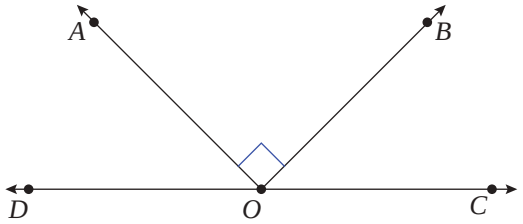
4. 

5. 

6. 

Use the definitions of acute, right, obtuse, and straight angles to answer each question.

7. In the figure shown, $\angle DOC$ is a straight angle and $m\angle BOA = 90°$.



a. What type of angle is $\angle AOC$?

b. What type of angle is $\angle BOC$?

c. What type of angle is $\angle BOA$?

8. Name the type of angle formed by the hands on a clock.

a. 
b. 
c. 
d. 

a. at six o'clock

b. at three o'clock

c. at one o'clock

d. at five o'clock

Use the definitions of complementary, supplementary, and straight angles to answer each question.

9. Assume that $\angle 1$ and $\angle 2$ are complementary.

a. If $m\angle 1 = 15°$, what is $m\angle 2$?

b. If $m\angle 1 = 3°$, what is $m\angle 2$?

c. If $m\angle 1 = 45°$, what is $m\angle 2$?

d. If $m\angle 1 = 75°$, what is $m\angle 2$?

10. Assume $\angle 3$ and $\angle 4$ are supplementary.

a. If $m\angle 3 = 45°$, what is $m\angle 4$?

b. If $m\angle 3 = 90°$, what is $m\angle 4$?

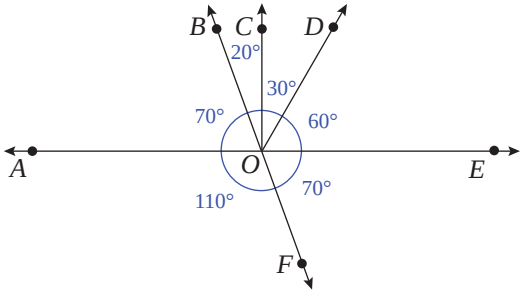
c. If $m\angle 3 = 110°$, what is $m\angle 4$?

d. If $m\angle 3 = 135°$, what is $m\angle 4$?

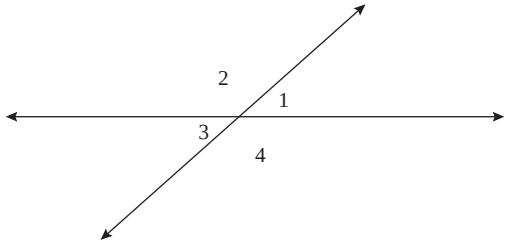
11. In the figure shown,

a. Name all of the pairs of supplementary angles.

b. Name all the pairs of complementary angles.



Use the definitions of adjacent and vertical angles to answer each question.



12. The figure shows two intersecting lines.

a. If $m\angle 1 = 30°$, what is $m\angle 2$?

b. Is $m\angle 3 = 30°$? Give a reason for your answer other than the fact that $\angle 1$ and $\angle 3$ are vertical angles.

c. Name two pairs of congruent angles.

d. Name four pairs of adjacent angles.

13. The figure shows two intersecting lines where $m\angle 1 = 30°$. Find the measures of the other three angles.

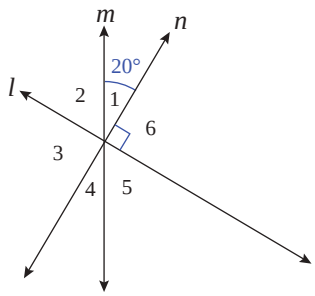
14. Given that $m\angle 1 = 42°$ in the figure, find the measures of the other three angles.

15. In the figure shown, *l*, *m*, and *n* are straight lines with $m\angle 1 = 20°$ and $m\angle 6 = 90°$.

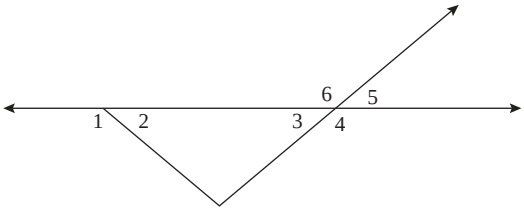
a. Find the measures of the other four angles.

b. Which angle is supplementary to $\angle 6$?

c. Which angles are complementary to $\angle 1$?

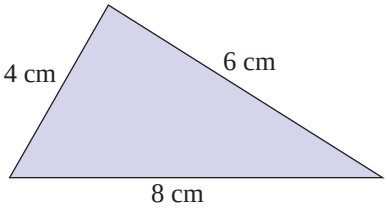


16. In the figure, $m\angle 2 = m\angle 3 = 40°$. Find all other pairs of angles that are congruent.



Classify each triangle in the most precise way possible, given the indicated lengths of its sides and/or measures of its angles.

17.



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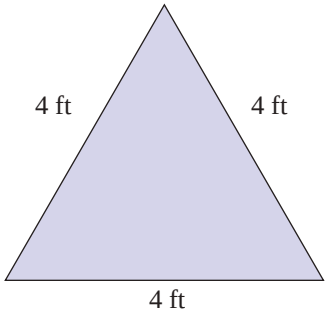

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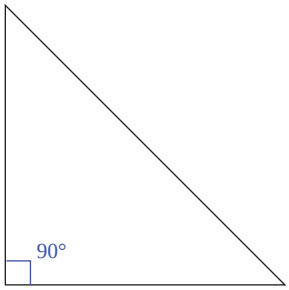

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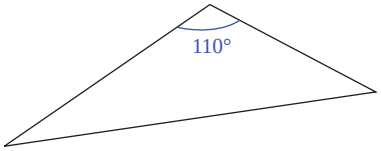

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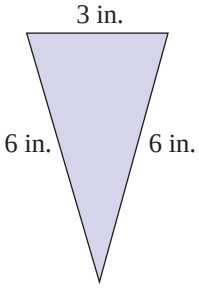

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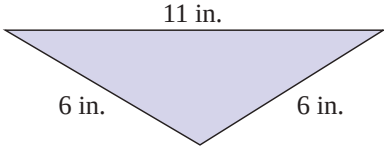

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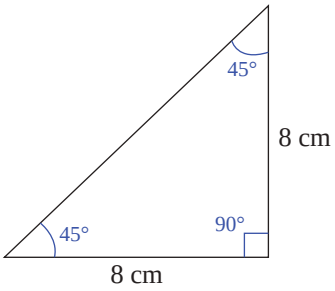

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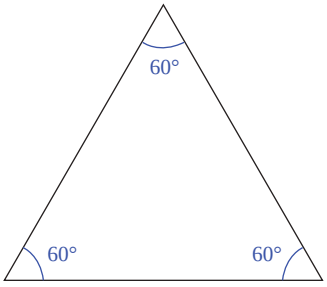

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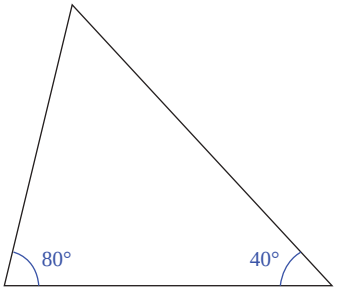

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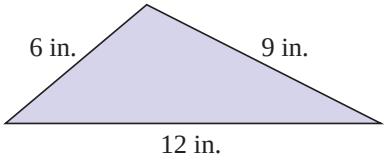

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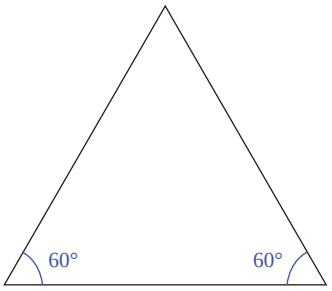

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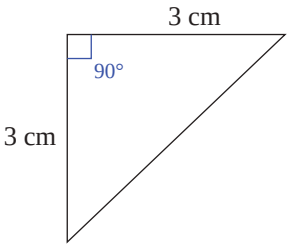
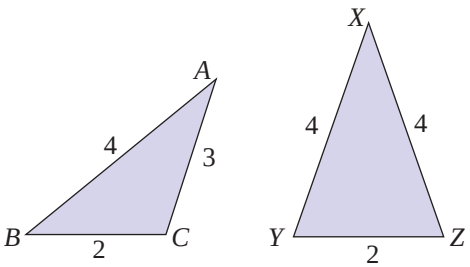


Determine whether each pair of triangles is similar. If the pair of triangles is similar, explain why and indicate the similarity by using the ~ symbol.

29.



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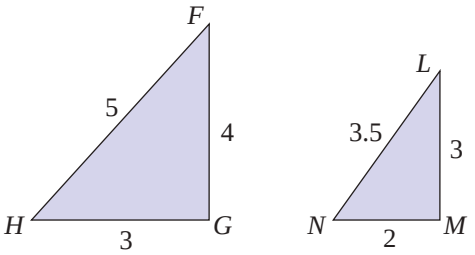

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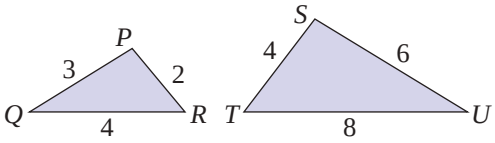

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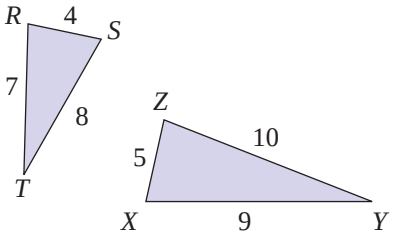

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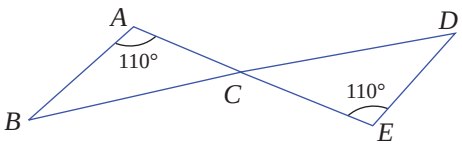

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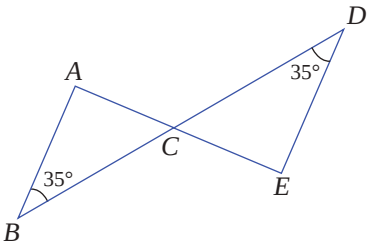
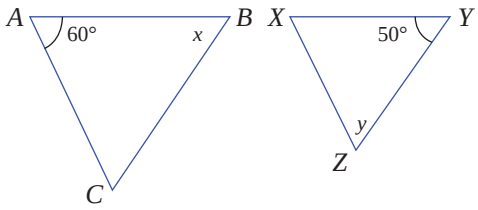


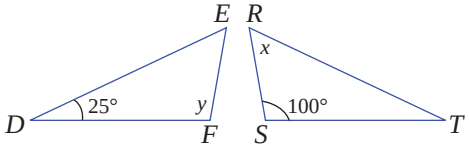
Find the values for x and y.

---

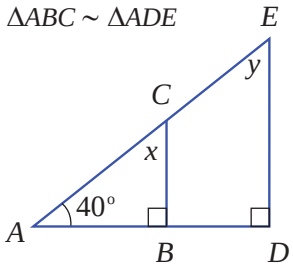
35. $\Delta ABC \sim \Delta XYZ$



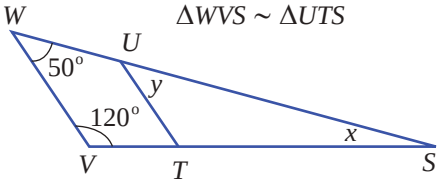
36. $\Delta DEF \sim \Delta TRS$



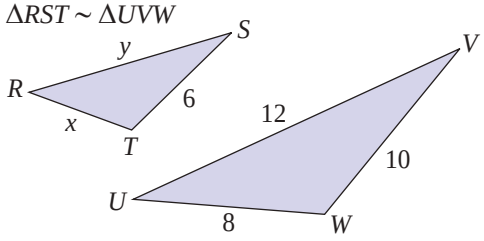
37. $\Delta ABC \sim \Delta ADE$



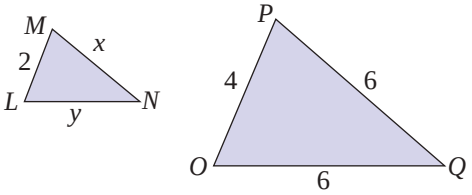
38. $\Delta WVS \sim \Delta UTS$



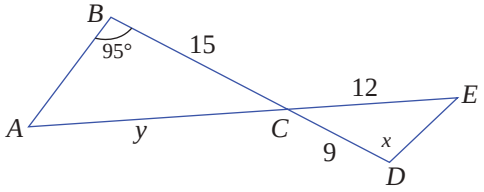
39. $\Delta RST \sim \Delta UVW$



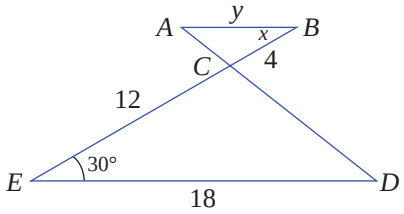
40. $\Delta LMN \sim \Delta OPQ$



41. $\Delta ABC \sim \Delta EDC$

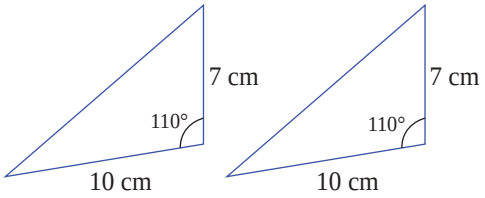


42. $\Delta ABC \sim \Delta EDC$



Determine whether each pair of triangles is congruent. If the pair of triangles is congruent, state the property that confirms that they are congruent.

43.



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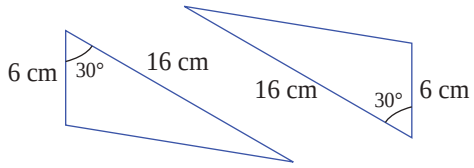

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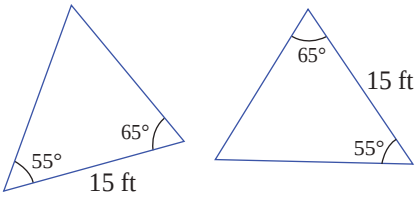

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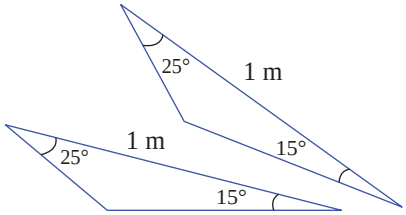

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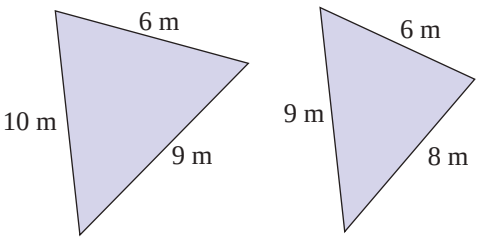

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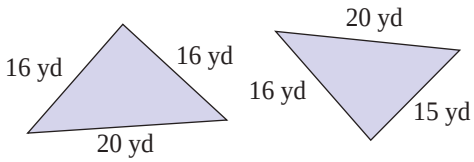

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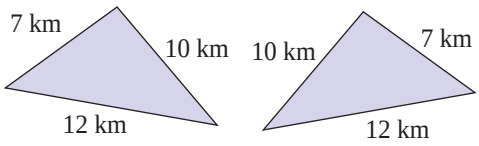

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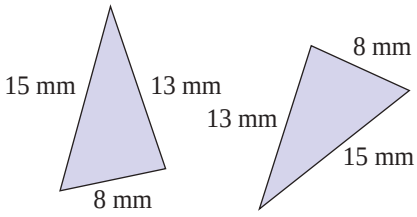
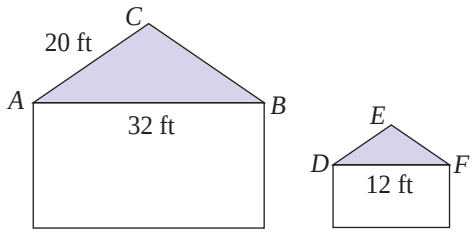


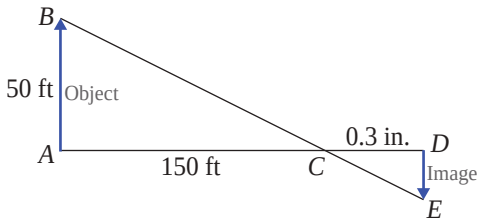
Applications

Solve.

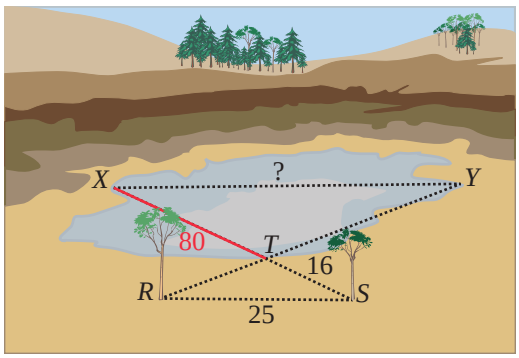
51. A child's playhouse is built to look like a smaller version of the family house, where the ends of the roofs have similar proportions. The width of the main house (AB) is 32 feet and the length from the peak to the gutter of the roof for one of the sides is 20 feet. If the width of the playhouse (DF) is 12 feet, what is the length from the peak to the gutter (DE) of the playhouse roof?



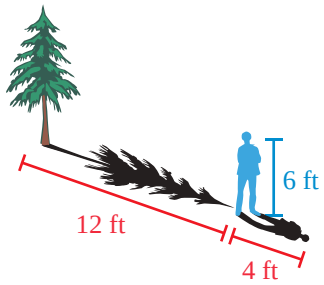
52. A camera uses a lens that will look at a properly focused object (such as a person or a tree) and then display an inverted image of this object on a screen or film that is on the opposite side of the lens as shown in the figure. If a picture of a 50-foot tall building (AB), which is 150 feet from the lens (AC) is photographed, how tall is the image (DE) if the film on the opposite side (CD) is 0.3 inch from the lens.



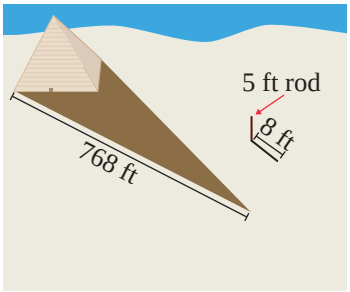
53. A surveyor is trying to figure out the length of a large quarry. The surveyor sees two trees at the edge of the quarry that are close together, and notices that the line between the two trees is parallel to the line that connects the edges of the quarry at its widest. We will call the two trees points R and S and the points X and Y will be the endpoints of the line connecting the two edges of the quarry. Drawing an imaginary line from R to Y and from S to X creates two similar triangles, ΔRST and ΔYXT, with a common point T. If the length of RS is 25 yd, the length of ST is 16 yd, and the length of XT is 80 yd, what is the length of the quarry (the distance between X and Y)? (See the figure.)



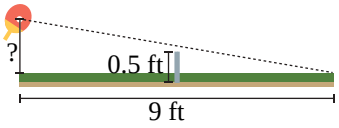
54. You and a friend are walking to class and want to figure out the height of the tree next to your building. Your friend is exactly 6 ft tall and casts 4 ft shadow. The tree casts a 12 ft shadow. How tall is the tree?



55. Thales was a mathematician circa 500 B.C. who wanted to know the height of the Great Pyramid. He discovered he could calculate the height of the pyramid using similar triangles. He stuck a rod in the ground that rose 5 ft into the air and cast an 8 ft shadow. If, at the same time, the length of the shadow from the Great Pyramid was 768 ft, determine the height of the Great Pyramid.



56. At what height off the table would you need to hit a ping pong ball for it to skim the net (0.5 feet tall) and hit on the edge of the opposite side of the table 9 feet away from you? Assume that the net is in the exact center of the table (with 4.5 feet on either side) and that your paddle is directly above the edge of the table.

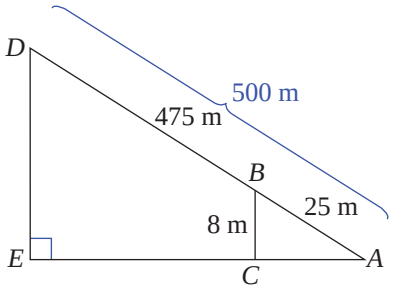


57. Your neighbors are hanging their holiday lights. The ladder they are currently using is 12 feet long and when leaned up against the house just reaches the top of their 8-foot tall porch. How long of a ladder will they need to reach the top of their chimney which is at a height of 32 feet? (Assume that both ladders are placed such that they make the same angle with the ground.)

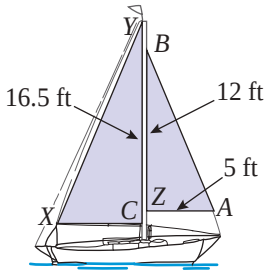
History

Thales was a Greek mathematician who lived approximately 624 BC to 540 BC. While he was an engineer by trade, he was considered by Aristotle to be the first philosopher in the Greek tradition. Thales has also been referred to as the Father of Science. While it is difficult to give Thales credit for measuring the Great Pyramid as described in Exercise 55 (due to a lack of written records), there are stories of him doing this as well as using this method to measure the distances of ships at sea.

58. The sloping surface of a hill (from base to peak) is 500 meters long. Paul starts at the base of the hill and walks uphill 25 meters which results in a gain of 8 meters in elevation. What is the height of the hill? (**Hint:** As shown on the accompanying drawing, there are two similar triangles ΔABC and ΔADE. Solve for DE.)



59. A sloop is a sailboat that has two triangular sails on a single mast. If the smaller sail is 12 feet along the mast (CB), and 5 feet along its bottom (AC), and the larger sail is 16.5 feet along the mast (ZY), how wide is the larger sail at the bottom (XZ) if ΔABC and ΔXYZ are similar triangles? Round your answer to the nearest tenth.



Writing & Thinking

60. **a.** The supplement of a right angle is what type of angle?

b. The supplement of an obtuse angle is what type of angle?

c. The supplement of an acute angle is what type of angle?

61. Determine the errors in the following statement. Assume $\Delta ABC \sim \Delta DEF$.

a. Corresponding angles are congruent. This means $m\angle A = m\angle D$, $m\angle B = m\angle F$, and $m\angle C = m\angle E$.

b. Corresponding sides are the same length.

62. Kelly needs to determine whether two triangles are similar. She was given the following information.

For ΔABC and ΔDEF, $AB = 3.6$, $AC = 2.4$, $BC = 2$ and $DE = 9$, $DF = 6$, $EF = 5$.

What should be her first step?