

## F Cost per Item

In the following example, note that the cost per item is found by dividing the total cost by the number of items sold. The cost per item changes because the total cost remains fixed while the number of items sold changes.

7. An artist has discovered that if she charges  $x$  dollars per painting, she can sell  $55 - x$  paintings in a month. What is the fewest number of paintings she can produce to make \$250 on her artwork in a month?

### Example 7 Application: Finding the Cost per Item

Carol has started selling bracelets online. She estimates that if she charges  $x$  dollars for each bracelet, she can sell  $19 - x$  bracelets per week. What is the smallest number of bracelets she can sell to make \$90 in one week?

#### Solution

Let  $x$  = price per bracelet

and  $19 - x$  = bracelets sold per week.

The total income is the price per object multiplied by the total number of objects sold.

$$x(19 - x) = 90$$

$$19x - x^2 = 90$$

$$0 = x^2 - 19x + 90$$

$$0 = (x - 10)(x - 9)$$

$$x - 10 = 0 \quad x - 9 = 0$$

$$x = 10 \quad \text{or} \quad x = 9$$

If Carol charges \$9 for each bracelet, she will have to sell  $19 - 9 = 10$  bracelets to make \$90 in one week. However, if she charges \$10 for each bracelet, she will have to sell  $19 - 10 = 9$  bracelets. Therefore, the smallest number of bracelets Carol can sell in a week and still make \$90 that week is 9 bracelets.

*Now work margin exercise 7.*

#### Margin Exercise Answers

1. 3 ft and 4 ft   2. 3 hours   3. 4.14 mph   4. In 30 seconds   5. 8 meters   6. 127.3 ft   7. 5 paintings

## 11.4 Exercises

### Concept Check

**Fill-in-the-Blank.** Complete each sentence using information found in this section.

- Application problems are designed to teach you to \_\_\_\_\_ carefully, to \_\_\_\_\_ clearly, and to \_\_\_\_\_ from English to algebraic expressions and equations.
- You must decide on a method of \_\_\_\_\_ based on the wording of the problem and your previous experience and knowledge.
- Draw a/an \_\_\_\_\_ for problems involving geometric figures whenever possible.

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

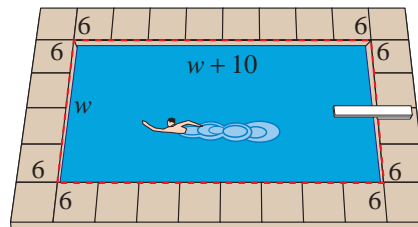
4. Application problems will always directly tell you which operations to perform.
5. The basic formula for distance-rate-time problems is  $d = rt$ .

## Applications

Solve.

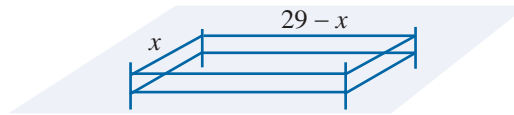
1. A rectangle has a length 5 m less than twice its width. If the area is  $63 \text{ m}^2$ , find the dimensions of the rectangle.
2. The length of a rectangle is 2 cm less than 3 times its width. If the area of the rectangle is  $225 \text{ cm}^2$ , find the dimensions of the original rectangle.
3. The difference between two positive numbers is 9. If the smaller number is added to the square of the larger number, the result is 147. Find the numbers.
4. The difference between a positive number and 3 is four times the reciprocal of the number. Find the number.  
(**Hint:** The reciprocal of  $x$  is  $\frac{1}{x}$ .)

5. The Wilsons have a rectangular swimming pool that is 10 ft longer than it is wide. The pool is completely surrounded by a concrete deck that is 6 ft wide. The total area of the pool and the deck is  $1344 \text{ ft}^2$ . Find the dimensions of the pool.



6. Each side of a square is increased by 10 cm. The area of the resulting square is 9 times the area of the original square. Find the length of the sides of the original square.
7. If 5 meters are added to each side of a square, the area of the resulting square is four times the area of the original square. Find the length of the sides of the original square.
8. The diagonal of a rectangle is 13 m. The length is 2 m more than twice the width. Find the dimensions of the rectangle.
9. The length of a rectangle is 4 m more than its width. If the diagonal is 20 m, what are the dimensions of the rectangle?
10. An orchard has 2030 trees. The number of trees in each row is 12 more than twice the number of rows. How many trees are in each row?
11. A rectangular auditorium seats 960 people. The number of seats in each row is 16 more than the number of rows. Find the number of seats in each row.
12. An apartment building has the same number of units on each floor. The building has five times as many units per floor as number of floors, and there are 405 units total. How many floors does the building have?

13. A farmer fenced in a 198-square-meter portion of his field with 58 meters of fencing. What are the length and width of the field? (**Hint:** The length plus the width is equal to half of the perimeter.)



- a. Write an equation to express the area of the fenced-in field.  
 b. Solve the equation from part a. for the variable.  
 c. Use the answer from part b. to determine the length and width of the fenced-in field.
14. A large U-Haul truck is 8 ft tall. The length of the truck is 4 ft longer than three times the width. What are the dimensions of the truck if the volume is 1590 ft<sup>3</sup>?

15. A photograph 9 in. wide and 12 in. long is surrounded by a frame of uniform thickness. The area of the frame itself, not including the center, is 162 in.<sup>2</sup> Find the thickness of the frame.

Area of Frame = 162 sq. in.



16. The Mona Lisa is a famous painting by Leonardo da Vinci. The painting is 30 in. by 21 in. It is surrounded by a frame of uniform thickness whose area (not including the center) is 756 in.<sup>2</sup> Find the thickness of the frame.
17. A 40-volt generator with a resistance of 4 ohms delivers power externally of  $40I - 4I^2$  watts, where  $I$  is the current measured in amperes. Find the current needed for the generator to deliver 100 watts of power.
18. Find the current needed for the 40-volt generator in Exercise 17 to deliver 64 watts of power.
19. Raymond operates a small sign-making business. He finds that if he charges  $x$  dollars for each sign, he sells  $40 - x$  signs per week. What is the least number of signs he can sell to have an income of \$336 in one week?
20. It costs Mrs. Snow \$3 to build a picture frame. She estimates that if she charges  $x$  dollars each, she can sell  $60 - x$  frames per week. What is the lowest price necessary to make a profit of \$432 each week?
21. Samuel operates a small peanut stand. He estimates that he can sell 600 bags of peanuts per day if he charges 50¢ for each bag. He determines that he can sell 20 more bags for each 1¢ reduction in price.
- a. What would be his revenue for one day if he charged 48¢ per bag?  
 b. What should he charge in order to have receipts of \$315?
22. A sporting goods store owner estimates that if he sells a certain model of basketball shoes for  $x$  dollars a pair, he will be able to sell  $125 - x$  pairs. Find the price if his sales are \$3750. Is there more than one possible answer?

23. Mr. Prince owns a 15-unit apartment complex. If all units are rented, the rent for each apartment is \$700 per month. Each time the rent is increased by \$70, he will lose 1 tenant. What is the rental rate if he receives \$10,920 monthly in rent? (**Hint:** Let  $x$  represent the number of empty units.)

24. The Ski Club is planning to charter a bus to a ski resort. The cost will be \$900 and each member will share the cost equally. If the club had 15 more members, the cost per person would be \$10 less. How many are in the club now? (**Hint:** If  $x$  = number in club now,  $\frac{900}{x}$  = cost per person.)



25. A motorboat takes a total of 2 hours to travel 8 miles downstream and 4 miles back on a river that is flowing at a rate of 2 mph. Find the rate of the boat in still water.

	Rate	Time	Distance
Going	$x$	?	200
Returning	$x - 10$	?	200

26. A small motorboat travels 12 mph in still water. It takes 2 hours longer to travel 45 miles going upstream than it does going downstream. Find the rate of the current. (**Hint:**  $12 + c$  = rate going downstream and  $12 - c$  = rate going upstream.)

27. Recently Mr. and Mrs. Roberts spent their vacation in San Francisco, which is 540 miles from their home. Being a little reluctant to return home, the Roberts took 2 hours longer on their return trip and their average speed was 9 mph slower than when they were going. What was their average rate of speed as they traveled from home to San Francisco?

28. Lisa traveled to a college that is located 200 miles from the city where she works to train customers how to use the software that her company sells. Due to a traffic jam, her average speed returning was 10 miles per hour less than her average speed going to the college. The total travel time to and from the college was 9 hours. What was Lisa's average speed going to the college?

- a. Use the table to set up a rational equation to describe the situation. Use the variable  $x$  to represent the average speed going to the college. (**Hint:** The sum of the times that it took Lisa to travel to and from the college is 9 hours.)

Distance ( $d$ )	÷	Rate ( $r$ )	=	Time ( $t = \frac{d}{r}$ )
Going				
Returning				


- b. Solve the equation from part a. Round your answer to the nearest tenth.  
 c. Which solution from part b. makes sense in the context of the situation? Explain your reasoning.  
 d. Use the answer from part c. to answer the question from the problem statement.

29. The Blumin Garden Club planned to give their president a gift of appreciation costing \$120 and to divide the cost evenly. In the meantime, 5 members dropped out of the club. If it now costs each of the remaining members \$2 more than originally planned, how many members initially participated in the gift buying?  
(**Hint:** If  $x$  = number in club initially,  $\frac{120}{x}$  = cost per member.)
30. A manufacturing crew needs to assemble 1000 boxes per day, divided equally among the workers. One day, three workers call in sick, and the remaining members each need to assemble 75 more boxes than usual. How many workers are on the manufacturing crew?
31. A toy football launcher can be used to launch a football straight up from a height of one foot at an initial velocity of 60 feet per second. The height of the ball can be found by using the equation  $h = -16t^2 + 60t + 1$ . When will the football hit the ground? Round your answer to the nearest tenth.
32. A bullet is fired straight up from a shoulder-height of 5 feet with a muzzle velocity of 360 feet per second. The height of the bullet can be found by using the equation  $h = -16t^2 + 360t + 5$ . When will the bullet hit the ground? Round your answer to the nearest tenth.
33. A woman and her daughter can paint their cabin in 3 hours. Working alone it would take the daughter 8 hours longer than it would the mother. How long would it take the mother to paint the cabin alone?
34. Two employees together can prepare a large order in 2 hrs. Working alone, one employee takes three hours longer than the other. How long does it take each person working alone?
35. Two pipes can fill a tank in 8 minutes if both are turned on. If only one is used it would take 30 minutes longer for the smaller pipe to fill the tank than the larger pipe. How long will it take the smaller pipe to fill the tank?
36. A farmer and his son can plow a field with two tractors in 4 hours. If it would take the son 6 hours longer than the father to plow the field alone, how long would it take each if they worked alone?
37. Jack and Diane are decorating a nursery room for their baby, who will be born in a few months. Working together, they can completely decorate the nursery in 4 hours. Working alone, it would take Diane 6 hours longer to decorate the nursery than it would take Jack. How long would it take Jack and Diane to decorate the nursery by themselves?

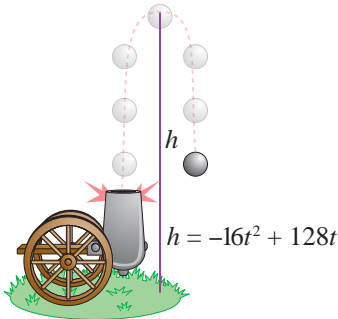
- a. Use the table to set up a rational equation to describe the situation. Use the variable  $x$  to represent the time it takes Jack to decorate the nursery by himself.

Person(s)	Time of Work (in Hours)	Part of Work Done in 1 Hour
Jack		
Diane		
Together		


- b. Solve the equation from part a.
- c. Which solution from part b. makes sense in the context of the situation? Explain your reasoning.
- d. Use the answer from part c. to answer the question from the problem statement.

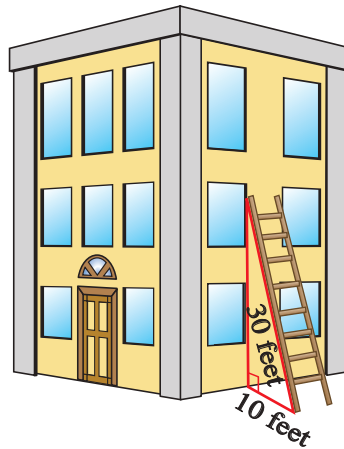
38.  A ball is thrown upward with an initial velocity of 32 ft/sec from the edge of a cliff near the beach. The cliff is 50 ft above the beach. The height of the ball can be found by using the equation  $h = -16t^2 + 32t + 50$ , where  $t$  is measured in seconds.
- When will the ball be 66 feet above the beach?
  - When will the ball be 30 feet above the beach?
  - In about how many seconds will the ball hit the beach?


39. The height of a projectile fired upward from the ground with a velocity of 128 ft/sec is given by the formula  $h = -16t^2 + 128t$ .

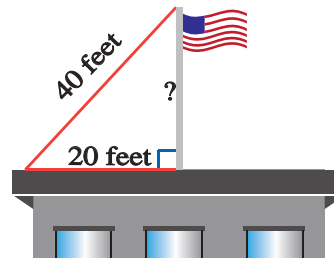



- When will the projectile be 256 feet above the ground?
- Will the projectile ever be 300 feet above the ground? Explain.
- When will the projectile be 240 feet above the ground?
- In how many seconds will the projectile hit the ground?

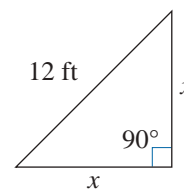
40.  A ladder is 30 ft long and you want to place the base of the ladder 10 ft from the base of a building. About how far up the building (to the nearest tenth of a foot) will the ladder reach?



41.  A flag pole is on top of a building and is held in place by steel cables attached to the top of the pole. If one such cable is 40 ft long and is attached at a point on the roof of the building 20 ft from the base of the flag pole, what is the length of the flag pole (to the nearest tenth of a foot)?



42.  A landscaper was given the task to create a triangular flower garden in the corner of an office building. The landscaper has 12 feet of low fencing to use as a border along one side of the garden. The final garden will have the shape shown in the figure. The landscaper needs to know the remaining side lengths of the triangle to determine the area he will need to cover with fresh topsoil.



- a. Use the Pythagorean Theorem to set up an equation which describes the relationship between the side lengths of the flower garden.
- b. Solve the equation from part a. for the variable. Round your answer(s) to the nearest tenth.
- c. Which solution from part b. makes sense in the context of the situation? Explain your reasoning.
- d. Use the answer from part c. to determine the area that the landscaper will need to cover with topsoil.

### Writing & Thinking

43. Suppose that you are to solve an applied problem and the solution leads to a quadratic equation. You decide to use the quadratic formula to solve the equation. Explain what restrictions you must be aware of when you use the formula.