$$\left(x-3-\sqrt{5}\right)\left(x-3+\sqrt{5}\right)=0$$
$$\left[(x-3)-\sqrt{5}\right]\left[(x-3)+\sqrt{5}\right]=0$$
$$(x-3)^2-\left(\sqrt{5}\right)^2=0$$
$$x^2-6x+9-5=0$$
$$x^2-6x+4=0$$

Regroup the terms to make the multiplication easier.

This equation has two solutions:
$x=3+\sqrt{5}$ and $x=3-\sqrt{5}$

Now work margin exercise 9.

Completion Example Answers

6.
$$2x^2-12x=\underline{-2}$$
$$x^2-6=\underline{-1}$$
$$x^2-6x+\underline{9}=\underline{-1}+\underline{9}$$
$$(x-\underline{3})^2=\underline{8}$$
$$x-\underline{3}=\pm\underline{\sqrt{8}}$$
$$x=\underline{3}\pm\underline{2\sqrt{2}}$$

Margin Exercise Answers

1. a. $y^2-14y+\underline{49}=\left(\underline{y-7}\right)^2$ **b.** $x^2+9x+\underline{\frac{81}{4}}=\left(\underline{x+\frac{9}{2}}\right)^2$ **2.** $x=5\pm2\sqrt{14}$ **3.** $x=-1\pm\sqrt{7}$ **4.** $x=\frac{-1\pm\sqrt{19}}{2}$ **5.** $x=-2\pm\sqrt{7}$ **6.** $x=1\pm\sqrt{6}$ **7.** $y^2-32=0$ **8.** $x^2-4x-14=0$ **9.** $x^2-2x-4=0$

11.2 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. Completing the square is the process of adding terms to binomials so that the result will be a perfect square ______.

2. To solve a quadratic equation by completing the square, arrange terms with _________ on one side of the equation and _________ on the other.

3. When solving by completing the square, the quadratic equation should have a leading coefficient of ______.

4. After finding the coefficient that completes the square of the polynomial, ______ this constant to both sides of the equation.

5. When completing the square, the constant term is the ______ of $\frac{1}{2}$ of the coefficient of x.

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True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

6. To get a leading coefficient of 1, multiply both sides of the equation by the reciprocal of the leading coefficient.

7. When solving a quadratic equation, there is either no solution or two solutions.

8. The last step of solving a quadratic equation by completing the square is to use the square root property.

Practice

Add the correct constant to complete the square; then factor the trinomial as indicated. See Example 1.

1. $x^2 - 12x + \_\_\_ = (\_\_\_\_\_)^2$

2. $y^2 + 14y + \_\_\_ = (\_\_\_\_\_)^2$

3. $x^2 + 6x + \_\_\_ = (\_\_\_\_\_)^2$

4. $x^2 + 8x + \_\_\_ = (\_\_\_\_\_)^2$

5. $x^2 - 5x + \_\_\_ = (\_\_\_\_\_)^2$

6. $x^2 + 7x + \_\_\_ = (\_\_\_\_\_)^2$

7. $y^2 + y + \_\_\_ = (\_\_\_\_\_)^2$

8. $x^2 + \frac{1}{2}x + \_\_\_ = (\_\_\_\_\_)^2$

9. $x^2 + \frac{1}{3}x + \_\_\_ = (\_\_\_\_\_)^2$

10. $y^2 + \frac{3}{4}y + \_\_\_ = (\_\_\_\_\_)^2$

11. $2x^2 + 4x + \_\_\_ = 2(\_\_\_\_\_)^2$

12. $3x^2 + 18x + \_\_\_ = 3(\_\_\_\_\_)^2$

Solve the quadratic equations by completing the square. See Examples 2 through 6.

13. $x^2 + 4x - 5 = 0$

14. $x^2 + 6x - 7 = 0$

15. $y^2 + 2y = 5$

16. $x^2 + 3 = 8x$

17. $x^2 + 3 = 10x$

18. $z^2 + 4z = 2$

19. $x^2 - 4x - 45 = 0$

20. $x^2 - 10x + 21 = 0$

21. $x^2 - 3x - 40 = 0$

22. $x^2 + x - 42 = 0$

23. $3x^2 + x - 4 = 0$

24. $2x^2 + x - 6 = 0$

25. $x^2 - 6x + 5 = 0$

26. $x^2 - 4x + 3 = 0$

27. $x^2 + 11 = 12x$

28. $x^2 = 6 - x$

29. $y^2 = 10y - 4$

30. $x^2 = 3 - 4x$

31. $z^2 + 3z - 5 = 0$

32. $x^2 - 5x + 5 = 0$

33. $x^2 + x - 1 = 0$

34. $y^2 + 3y + 1 = 0$

35. $x^2 + 5x + 2 = 0$

36. $4x^2 + 7x + 2 = 0$

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37. $3x^2 - 10x + 5 = 0$

38. $3y^2 + 5y - 3 = 0$

39. $3x^2 + 6x - 12 = 0$

40. $4x^2 + 8x - 8 = 0$

41. $-4x - 5 = -8x^2$

42. $2x - 2 = -6x^2$

43. $5x^2 + 15x - 25 = 0$

44. $4x^2 + 28x + 32 = 0$

45. $3y^2 = 4 - y$

46. $2x^2 + 4 = -9x$

47. $2x^2 - 8x + 4 = 0$

48. $3x^2 - 18x + 12 = 0$

Write a quadratic equation with integer coefficients that has the given roots. See Examples 7 through 9.

49. $x = \sqrt{7}, x = -\sqrt{7}$

50. $x = \sqrt{6}, x = -\sqrt{6}$

51. $x = 1 + \sqrt{3}, x = 1 - \sqrt{3}$

52. $z = 3 + \sqrt{2}, z = 3 - \sqrt{2}$

53. $y = -2 + 2\sqrt{5}, y = -2 - 2\sqrt{5}$

54. $x = 1 + 2\sqrt{3}, x = 1 - 2\sqrt{3}$

Applications

Solve.

55. A local frame shop determines that the revenue function for their custom framing service is $R(p) = 360p - 4p^2$, where p is the base price in dollars for each custom framing job.

a. Set the function equal to 0 and solve for p using the method of completing the square.

b. What do the solutions from part a. mean?

56. The height of a golf ball that is hit from the ground at a speed of 128 feet per second can be modeled with the expression $h(t) = -16t^2 + 128t$, where t is the time in seconds after the ball is hit.

a. Set the function equal to 0 and solve for t using the method of completing the square.

b. What do the solutions from part a. mean?

Writing & Thinking

57. Explain, in your own words, the steps involved in the process of solving a quadratic equation by completing the square.

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