10.6 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. The objective in simplifying a rational fraction is to find an equivalent fraction that has no radicals in the ______________.

2. To rewrite a fraction without irrational numbers in the denominator is to ______________ the denominator.

3. Calculations of sums and differences are much easier if the denominators are ______ expressions.

4. The product of the conjugates $a + b$ and $a - b$ is the difference of two ______________.

5. To rationalize a denominator containing a sum or difference that involves a square root, multiply both the numerator and the denominator by the ______________ of the denominator.

6. The conjugate of __________ is $6+\sqrt{x}$.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The conjugate of $y-\sqrt{5}$ is $y+\sqrt{5}$.

8. To rationalize the denominator, multiply only the denominator by an expression that will result in a denominator with no radicals.

9. To rationalize a fraction whose denominator is $\sqrt[3]{a}$, you would need to multiply the numerator and the denominator by $\sqrt[3]{a}$.

10. The fraction $\frac{\sqrt{2}}{3}$ is in simplest form.

Practice

Rationalize the denominator and simplify, if possible. Assume that all variables represent positive real numbers.

1. $\frac{5}{\sqrt{2}}$

2. $\frac{7}{\sqrt{5}}$

3. $\frac{-3}{\sqrt{7}}$

4. $\frac{-10}{\sqrt{2}}$

5. $\frac{6}{\sqrt{3}}$

6. $\frac{8}{\sqrt{2}}$

7. $\frac{\sqrt{18}}{\sqrt{2}}$

8. $\frac{\sqrt{25}}{\sqrt{3}}$

9. $\frac{\sqrt{27x}}{\sqrt{3x}}$

10. $\frac{\sqrt{45y}}{\sqrt{5y}}$

11. $\frac{\sqrt{ab}}{\sqrt{9ab}}$

12. $\frac{\sqrt{5}}{\sqrt{12}}$

13. $\dfrac{\sqrt{4}}{\sqrt{3}}$

14. $\sqrt{\dfrac{3}{8}}$

15. $\sqrt{\dfrac{9}{2}}$

16. $\sqrt{\dfrac{3}{5}}$

17. $\sqrt{\dfrac{1}{x}}$

18. $\sqrt{\dfrac{x}{y}}$

19. $\sqrt{\dfrac{2x}{y}}$

20. $\sqrt{\dfrac{x}{4y}}$

21. $\dfrac{2}{\sqrt{2y}}$

22. $\dfrac{-10}{3\sqrt{5}}$

23. $\dfrac{21}{5\sqrt{7}}$

24. $\dfrac{x}{5\sqrt{x}}$

25. $\dfrac{-2y}{5\sqrt{2y}}$

26. $\dfrac{\sqrt[3]{35}}{\sqrt[3]{4}}$

27. $\dfrac{\sqrt[3]{10}}{\sqrt[3]{9}}$

28. $-\sqrt{\dfrac{2}{3y}}$

29. $-\sqrt{\dfrac{25}{x^3}}$

30. $\dfrac{\sqrt{8x}}{\sqrt{5y^2}}$

31. $\dfrac{\sqrt{4x}}{\sqrt{3y^2}}$

32. $\dfrac{\sqrt{16y^2}}{\sqrt{2y^3}}$

33. $\dfrac{\sqrt{24b}}{\sqrt{6b^2}}$

34. $\sqrt[3]{\dfrac{2y^3}{27x^2}}$

35. $\sqrt[3]{\dfrac{7x}{2y^4}}$

36. $\dfrac{\sqrt[3]{6a^4}}{\sqrt[3]{25a^2b^4}}$

37. $\dfrac{\sqrt[3]{x^5}}{\sqrt[3]{9xy}}$

38. $\dfrac{\sqrt[3]{24x}}{\sqrt[3]{9}}$

39. $\sqrt[3]{\dfrac{11}{4}}$

40. $\sqrt[3]{\dfrac{3x^3}{8y^2}}$

41. $\dfrac{\sqrt[3]{5a}}{\sqrt[3]{3b^4}}$

42. $\sqrt[3]{\dfrac{7x^4}{16x^2y^4}}$

43. $\dfrac{\sqrt[3]{a^5}}{\sqrt[3]{4ab}}$

44. $\dfrac{3}{1+\sqrt{2}}$

45. $\dfrac{2}{\sqrt{6}-2}$

46. $\dfrac{-11}{\sqrt{3}-4}$

47. $\dfrac{1}{\sqrt{5}-3}$

48. $\dfrac{7}{3-2\sqrt{2}}$

49. $\dfrac{-6}{5-3\sqrt{2}}$

50. $\dfrac{11}{2\sqrt{3}+1}$

51. $\dfrac{-\sqrt{3}}{\sqrt{2}+5}$

52. $\dfrac{\sqrt{2}}{\sqrt{7}+4}$

53. $\dfrac{7}{1-3\sqrt{5}}$

54. $\dfrac{-3\sqrt{3}}{6+\sqrt{3}}$

55. $\dfrac{1}{\sqrt{3}-\sqrt{5}}$

56. $\dfrac{-4}{\sqrt{7}-\sqrt{3}}$

57. $\dfrac{-5}{\sqrt{2}+\sqrt{3}}$

58. $\dfrac{7}{\sqrt{2}+\sqrt{5}}$

59. $\dfrac{4}{\sqrt{x}+1}$

60. $\dfrac{-7}{\sqrt{x}-3}$

61. $\dfrac{5}{6+\sqrt{y}}$

62. $\dfrac{x}{\sqrt{x}+2}$

63. $\dfrac{8}{2\sqrt{x}+3}$

64. $\dfrac{3\sqrt{x}}{\sqrt{2x}-5}$

65. $\dfrac{\sqrt{4y}}{\sqrt{5y}-\sqrt{3}}$

66. $\dfrac{\sqrt{3x}}{\sqrt{2}+\sqrt{3x}}$

67. $\dfrac{3}{\sqrt{x}-\sqrt{y}}$

68. $\dfrac{4}{2\sqrt{x}+\sqrt{y}}$

69. $\dfrac{x}{\sqrt{x}+2\sqrt{y}}$

70. $\dfrac{y}{\sqrt{x}-\sqrt{3y}}$

71. $\dfrac{\sqrt{3}+1}{\sqrt{3}-2}$

72. $\dfrac{\sqrt{2}+4}{5-\sqrt{2}}$

73. $\dfrac{\sqrt{5}-2}{\sqrt{5}+3}$

74. $\dfrac{1+\sqrt{3}}{3-\sqrt{3}}$

75. $\dfrac{\sqrt{x}+1}{\sqrt{x}-1}$

76. $\dfrac{\sqrt{x}-4}{\sqrt{x}+3}$

77. $\dfrac{\sqrt{x}+2}{\sqrt{3x}+y}$

78. $\dfrac{3-\sqrt{x}}{2\sqrt{x}+y}$

Identify the error(s) made in the following attempt to rationalize a denominator.

79. $$\frac{y}{\sqrt{3}+y}=\frac{y}{\sqrt{3}+y}\cdot\frac{\sqrt{3}+y}{\sqrt{3}+y}=\frac{y\left(\sqrt{3}+y\right)}{3+y^2}=\frac{y\sqrt{3}+y^2}{3+y^2}$$

Applications

Solve.

80. Officers often need to recreate events that happen during accidents while they investigate, especially determining the initial speed of a car at the time of an accident. One way to do this is to use the formula $\frac{s}{\sqrt{l}}=k,$ where s is the initial speed of the vehicle in mph, l is the length of the skid marks left in feet, and k is a constant that depends on the driving conditions at the time of the accident.

a. Rationalize the denominator of the formula.

b. A driver claims that he was driving the speed limit, 55 mph, at the time of an accident. The skid marks on the road measured 176 feet. Officers estimate that the driving condition constant k based on the conditions at the time of the accident is $\sqrt{24}.$ Based on the formula, is the driver's claim correct? If not, what was the driver's initial speed before the accident?

81. The radius of a cylinder can be expressed in terms of its volume and its height by $r=\sqrt{\frac{V}{\pi h}}.$ Rationalize the denominator of this formula.

82. A company that sells computers learns that their income can be represented by the equation $I=\frac{6500p}{\sqrt{p}-10}$ dollars when they sell their computers for p dollars. Rationalize the denominator of this equation.

83. An intern at NASA needs to construct a cylinder to be used as a fuel cell for a scale model of a rocket. Her instructions are to make a fuel cell with a volume of $200\pi\text{ cm}^3$. To find the radius of the fuel cell for a certain height h and volume V, she uses the equation $r=\sqrt{\frac{V}{h\pi}}.$ Keep all answers in simplified radical form.

a. Find the radius of the fuel cell if the height is 12 cm.

b. Find the radius of the fuel cell if the height is 15 cm.

c. Find the radius of the fuel cell if the height is 24 cm.

84. The formula $r = \sqrt{\frac{A}{P}} - 1$ is used to determine the interest rate r of on an investment of initial value P that has a value of A after two years.

a. Rationalize the denominator of the fraction in the formula.

b. Find the interest rate on an investment with initial value \$1000 that has a value of \$1102.50 after 2 years.

85. A client tells his financial consultant that he has \$6000 to invest and would like to earn \$615 on his investment after 2 years. The client needs to know what the average interest rate of the investment will need to be to meet his expectations. The financial consultant can use the formula $A = P(r+1)^2$ to find the future amount A of an investment with a starting principle P and interest rate r after 2 years.

a. Solve the equation for r. Be sure to rationalize any denominators. (**Hint:** Divide both sides by P first and then take the square root of each side.)

b. Determine the interest rate that the \$6000 would need to be invested at to meet the client's expectations. Be sure to express the rate as a percent.

Writing & Thinking

86. In your own words, explain how to rationalize the denominator of a fraction containing the sum or difference of square roots in the denominator. Why does this work?