

D Evaluating Radical Expressions with a Graphing Calculator

Techniques for using a TI-84 Plus graphing calculator to evaluate radical expressions were illustrated in Section 10.1. These same basic techniques are used to evaluate numerical expressions that contain sums, differences, products, and quotients of radicals. Be careful to use parentheses to ensure that the rules for order of operations are maintained. In particular, sums and differences in numerators and denominators of fractions must be enclosed in parentheses. Study the following example carefully.

Example 5 Using a Calculator to Evaluate Radical Expressions

Use a TI-84 Plus graphing calculator to evaluate each expression. Round answers to the nearest ten-thousandth.

a. $3 + 2\sqrt{5}$

b. $(\sqrt{2} + 5)(\sqrt{2} - 5)$

Solution

a. The display should appear as follows.

Thus, $3 + 2\sqrt{5} = 7.4721359\dots \approx 7.4721$.

Rounded to the nearest ten-thousandth

b. The display should appear as follows.

Thus, $(\sqrt{2} + 5)(\sqrt{2} - 5) = -23$.

Note: When entering $\sqrt{2}$ on a calculator, the right parenthesis after the 2 must be included. Otherwise, the calculator will interpret the expression as $\sqrt{(2+5)}$ (or $\sqrt{7}$) which is not intended.

5. Use a TI-84 Plus graphing calculator to evaluate each expression. Round answers to the nearest ten-thousandth.

a. $4 + 5\sqrt{2}$

b. $(\sqrt{8} - 1)(\sqrt{8} + 1)$

Now work margin exercise 5.

Completion Example Answers

4. a. $6\sqrt{3} \cdot \sqrt{3} + 6\sqrt{3} \cdot 2\sqrt{7} = 18 + 12\sqrt{21}$ b. $\sqrt{x} \cdot \sqrt{x} + \sqrt{x} \cdot (-4) + 8 \cdot \sqrt{x} + 8 \cdot (-4) = x + 4\sqrt{x} - 32$

Margin exercise answers:

1. a. $\sqrt{66}$ b. $5\sqrt{7}$ c. $5y\sqrt{3}$ 2. a. $7 + 7\sqrt{3}$ b. $27 - 17\sqrt{5}$ c. $7a - 9$ 3. a. $4\sqrt{30} - 20$

b. $s + 2\sqrt{s} - 63$ 4. a. $\frac{\sqrt{3}}{5}$ b. $7\sqrt{4x+6}$ c. $7\sqrt{4x+6}$ 5. a. 11.0711 b. 7

10.5 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

- To find the product of radicals, we proceed just as in multiplying polynomials and use the _____ property.

- To find the product of two binomials that contain radical terms, you can use the _____ method.
- When translating English phrases into radical expressions, a/an _____ can indicate a change in operation.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

- The multiplication property of radicals says that $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$ for all values of a and b .
- The radicals do not need to be like radicals when multiplying radical expressions.
- The keywords *product* and *triple* refer to multiplication.


Practice

Multiply the following radical expressions and then simplify the results. Assume that all variables represent positive real numbers. See Examples 1, 2, and 3.

- $\sqrt{2}(3 - 4\sqrt{2})$
- $2\sqrt{7}(\sqrt{7} + 3\sqrt{2})$
- $3\sqrt{18} \cdot \sqrt{2}$
- $2\sqrt{10} \cdot \sqrt{5}$
- $-2\sqrt{6} \cdot \sqrt{8}$
- $2\sqrt{15} \cdot 5\sqrt{6}$
- $\sqrt{3}(\sqrt{2} + 2\sqrt{12})$
- $\sqrt{2}(\sqrt{3} - \sqrt{6})$
- $\sqrt{y}(\sqrt{x} + 2\sqrt{y})$
- $\sqrt{x}(\sqrt{x} - 3\sqrt{y})$
- $(3 + \sqrt{2})(5 - \sqrt{2})$
- $(\sqrt{6} + 2)(\sqrt{6} - 2)$
- $(\sqrt{3x} - 8)(\sqrt{3x} - 1)$
- $(6 + \sqrt{2x})(4 + \sqrt{2x})$
- $(2\sqrt{7} + 4)(\sqrt{7} - 3)$
- $(5\sqrt{3} - 2)(2\sqrt{3} - 7)$
- $(\sqrt{5} + 2\sqrt{2})^2$
- $(2\sqrt{5} + 3\sqrt{2})^2$
- $(\sqrt{2} + \sqrt{3})(\sqrt{5} - \sqrt{3})$
- $(\sqrt{6} + \sqrt{5})(\sqrt{6} - \sqrt{2})$
- $(\sqrt{x} + \sqrt{6})(\sqrt{x} - 3\sqrt{6})$
- $(\sqrt{11} + \sqrt{3})(\sqrt{11} - 2\sqrt{3})$
- $(3\sqrt{7} + \sqrt{5})(3\sqrt{7} - \sqrt{5})$
- $(7\sqrt{x} + \sqrt{2})(7\sqrt{x} - \sqrt{2})$
- $(\sqrt{x} + 5\sqrt{y})^2$
- $(3\sqrt{x} + \sqrt{y})^2$
- $(\sqrt{x+3} - 5)^2$
- $(\sqrt{x+2} + 3)^2$
- $(4 - \sqrt{2x+3})^2$
- $(6 - \sqrt{4x+1})^2$

Translate the given phrase into an algebraic expression. Do not simplify. See Example 4.

31. A radicand of five with an index of two
32. The cube root of twelve
33. Four times the square root of ten x
34. The square root of five, divided by twelve x
35. The cube root of six y , plus seven y
36. The difference between eight x and the square root of eight x
37. The sum of fifteen and four times the square root of x
38. The product of two and the cube root of six y , plus eleven y
39. The cube root of four minus eight b
40. The cube root of x , divided by x minus three
41. The quotient of the square root of three and a , minus seven
42. Seven times the square root of y , divided by one minus y

 Use a graphing calculator to evaluate each expression. Round your answers to the nearest ten-thousandth, if necessary. See Example 5.

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|---------------------------------|--|
| 43. $13 - \sqrt{75}$ | 47. $(\sqrt{7} + 8)(\sqrt{7} - 8)$ |
| 44. $5 - \sqrt{67}$ | 48. $(\sqrt{8} - \sqrt{5})(\sqrt{8} + \sqrt{5})$ |
| 45. $\sqrt{900} + \sqrt{2.56}$ | 49. $(2\sqrt{3} + 5\sqrt{2})(\sqrt{10} - 3\sqrt{5})$ |
| 46. $\sqrt{1600} - \sqrt{1.69}$ | 50. $(6\sqrt{5} + 5\sqrt{7})(3\sqrt{2} - \sqrt{6})$ |

Applications

Solve.

51. The city planning committee is looking for places to build a community garden.

One lot up for consideration has a length of $\frac{8 + 2\sqrt{b}}{a}$ and a width of $\frac{6 + 5\sqrt{b}}{a}$.

If $a = 3$ yards and $b = 7$ yards, what is the area of this lot? Leave your answer in simplified radical form.

52. The owner of an apple orchard has a field of new trees that are growing and producing more apples each year. He wants to determine the average growth rate, or percentage increase, in the amount of apples produced by these trees over time. The amount of growth varies each year, so he decides to find the geometric mean, or average. The formula $g = \sqrt{a} \cdot \sqrt{b}$ is used to find the geometric mean of two numbers.
- a. Over two years, the growth rate was 180% and 120%. Find the average growth rate for these two years. Leave your answer in simplified radical form.
 - b. Over two years, the growth rate was $8x^3\%$ and $9x^5\%$. Find the average growth rate for these two years. Leave your answer in simplified radical form.