

Tests for Divisibility

As mentioned in the note earlier in the section, here are the quick tests for divisibility.

A number is divisible

By 2: if the units digit is 0, 2, 4, 6, or 8.

By 3: if the sum of the digits is divisible by 3.

By 4: if the number formed by the last two digits is divisible by 4.

By 5: if the units digit is 0 or 5.

By 6: if the number is divisible by both 2 and 3.

By 9: if the sum of the digits is divisible by 9.

By 10: if the units digit is 0.

PROCEDURE

Margin Exercise Answers

1. **a.** 13 has exactly two factors, 1 and 13. **b.** 19 has exactly two factors, 1 and 19. **2. a.** 1, 5, and 25 are all factors of 25. **b.** 1, 2, 4, 8, 16, and 32 are all factors of 32. **3.** $2^2 \cdot 5 \cdot 7$ **4. a.** $2 \cdot 37$
b. $-1 \cdot 2^2 \cdot 5^2$ **c.** $2 \cdot 3^2 \cdot 13$ **5.** 126 **6.** 420

1.2 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. Every counting number except the number 1 has at least _____ factors.
2. A _____ number is a counting number greater than 1 with exactly two different factors.
3. A number is _____ by another number if the remainder in the division process is 0.
4. If a whole number is not divisible by _____, then it is odd.
5. The _____ factorization of a number is the product of all prime factors of that number.
6. The least common multiple of two or more counting numbers is the _____ number that is a multiple of each of those numbers.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

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7. A composite number is a counting number with exactly two different factors.
 8. An even number is a number that is divisible by 2.
 9. Each composite number has exactly one prime factorization.
 10. A number is divisible by 3 if the units digit is divisible by 3.

Practice

Determine which numbers, if any, in each set of counting numbers are prime. See Example 1.

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|---------------------|-----------------------------|
| 1. {13, 15, 17, 21} | 3. {2, 4, 6, 8, 10, 12, 14} |
| 2. {11, 19, 23, 51} | 4. {7, 16, 25, 36, 47, 49} |

Find two factors of each number (other than 1 and the number itself) to determine that the number is composite. (Answers will vary.) See Example 2.

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|--------|---------|
| 5. 72 | 10. 417 |
| 6. 63 | 11. 170 |
| 7. 68 | 12. 99 |
| 8. 39 | 13. 444 |
| 9. 502 | 14. 230 |

Find the prime factorization of each of the numbers. If a number is prime, write "prime." See Examples 3 and 4.

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|----------|----------|
| 15. 52 | 25. 125 |
| 16. 60 | 26. 343 |
| 17. 616 | 27. -400 |
| 18. 460 | 28. -500 |
| 19. -308 | 29. 120 |
| 20. -155 | 30. 196 |
| 21. 79 | 31. 231 |
| 22. 43 | 32. 675 |
| 23. 289 | 33. 1692 |
| 24. 361 | 34. 1716 |

35. List the first ten **multiples** of each number.
- a. 5
 - b. 6
 - c. 10
 - d. 15
36. From the lists you made in Exercise 35, find the least common multiple for each of the following sets of numbers.
- a. {5, 6}
 - b. {6, 10}
 - c. {5, 10, 15}
 - d. {6, 10, 15}

Find the LCM of each of the following sets of counting numbers. See Examples 5 and 6.

- 37. {3, 5, 7}
- 38. {2, 7, 11}
- 39. {8, 10}
- 40. {9, 12}
- 41. {2, 3, 11}
- 42. {3, 5, 13}
- 43. {4, 14, 35}
- 44. {10, 12, 20}
- 45. {50, 75}
- 46. {30, 70}
- 47. {20, 90}
- 48. {50, 80}
- 49. {28, 98}
- 50. {45, 75}
- 51. {10, 15, 35}
- 52. {6, 24, 30}
- 53. {15, 45, 90}
- 54. {14, 28, 56}
- 55. {20, 50, 100}
- 56. {30, 60, 120}
- 57. {10, 15, 25}
- 58. {22, 44, 121}
- 59. {26, 28, 91}
- 60. {34, 51, 54}
- 61. {35, 40, 72}
- 62. {30, 35, 63}
- 63. {12, 21, 44}
- 64. {20, 28, 45}
- 65. {99, 121, 231}
- 66. {81, 225, 324}
- 67. {48, 120, 144, 192}
- 68. {125, 135, 225, 250}
- 69. {40, 56, 160, 196}
- 70. {35, 49, 63, 126}

Applications

Solve.

71. Two astronauts miss connections at their first meeting in space.
- a. If one astronaut circles the earth every 15 hours and the other every 18 hours, in how many hours will they meet again at the same place?
 - b. How many more orbits will each astronaut have to complete between missing their first meeting and making their second meeting?

72. Three neighbors mow their lawns at different intervals during the summer months. The first one mows every 5 days, the second every 7 days, and the third every 10 days.
- How frequently do they mow their lawns on the same day?
 - How many times does each neighbor mow in between the times when they all mow together?
73. Four women work for the same book company selling textbooks. They leave the home office on the same day and take 8 days, 12 days, 14 days, and 15 days, respectively, to visit schools in their own sales regions.
- In how many days will they all meet again at the home office?
 - How many sales trips will each have made in this time?
74. A fruit production company has three packaging facilities, each of which uses different-sized boxes as follows: 24 pieces/box, 36 pieces/box, and 45 pieces/box.
- Assuming that the truck provides the same quantity of uniformly-sized pieces of fruit to all three packaging facilities, what is the minimum number of pieces of fruit that will be delivered so that no fruit will be left over?
 - How many boxes will each facility package?
75. Three swimmers decide to swim laps together, and they will quit when they reach the starting end of the pool together. The first swimmer can swim a lap in 35 seconds, the second will take 40 seconds, and the third takes 42 seconds.
- How many seconds will it take before they quit?
 - How many laps will each swimmer swim in that interval?
76. Two analog clocks are sitting next to each other. The first clock keeps perfect time, where the minute hand takes 60 minutes to travel completely around the dial. The second clock runs fast and the minute hand makes one complete revolution in 55 minutes.
- Assuming that both clocks are started so that the minute hands are at 12, how many minutes will it take until both minute hands return to 12 at the same time?
 - How many hours does this represent?

Writing & Thinking

77. List five prime numbers larger than 50.
78. Describe, in your own words, how to find the LCM of a set of counting numbers.
79.
 - Explain why 1 is not a prime number.
 - Explain why 1 is not a composite number.