INTRODUCTORY ALGEBRA

CHAPTER PROJECTS

SEVENTH EDITION

D. FRANKLIN WRIGHT

HAWKES LEARNING

Chapter 1 Project

On a Budget

An activity to demonstrate the use of fractions in real life.

Samantha's friend Meghan is always traveling to exotic locations when she goes on vacation. One day, Samantha asked Meghan how she was able to afford it. Meghan told her it was simple: she makes a budget and sets aside a portion of her income each month for a vacation. Meghan told Samantha that they could meet at her house for dinner on Saturday and she would be glad to show her how to make a budget.

The table below shows the amount Meghan spends on each category in her budget. Assume Meghan makes $4400 a month (after taxes) and answer the following questions. Fractions should be written in reduced form.

Meghan's Budget

Category	Amount	Category	Amount
Rent	$1100	Car Loan	$528
Groceries	$352	IRA	$264
Utilities	$220	Charity	$440
Dining Out	$264	Savings	$275
Vacation	$440	Car Insurance	$176
Gas	$88	Phone	$110

1. Samantha earns a different amount per month and wants to compare her spending to Meghan's, so she decides to convert all of Meghan's spending to fractions. Find the fractional values for Meghan's categories by dividing each amount by her monthly income and reducing the fractions to lowest terms.

Meghan's Budget

Category	Amount	Category	Amount
Rent		Car Loan	
Groceries		IRA	
Utilities		Charity	
Dining Out		Savings	
Vacation		Car Insurance	
Gas		Phone	

2. Find the sum of the fractions in the budget. Begin by adding the fractions that have like denominators, then add the fractions by finding a common denominator.

3. Samantha tells Meghan that her budget must be incomplete. Meghan informs her that the remaining is what she spends on miscellaneous items like cleaning supplies, toiletries, and clothes. What fraction of her budget does Meghan spend on miscellaneous items?

4. Samantha determines that she spends $\frac{1}{10}$ of her income on groceries and $\frac{1}{8}$ dining out. What fraction of her income in spent on food?

5. What fraction of Meghan's income is spent on food?

6. Find the difference between the fractional amount that Meghan and Samantha spend for food. Who uses the largest fractional part of their income for food?

7. Add the fractional values for the car loan, car insurance, and gas to determine the fractional amount of Meghan's income that is set aside for her car.

8. Samantha doesn't have a car payment, but she has to pay for parking at her apartment. She has determined that she spends $\frac{1}{50}$ of her monthly income on car insurance, $\frac{3}{50}$ on parking, and $\frac{1}{25}$ on gas. What fractional part of her income is used for her car?

9. Find the difference between the fractional amount that Meghan and Samantha spend on their cars. Who uses the largest fractional part of their income for car expenses?

10. Meghan plans to move into a new apartment with a roommate and travel more often. She needs to revise her budget accordingly.

Meghan's Budget

Category	Amount	Category	Amount
Rent		Car Loan	
Groceries		IRA	
Utilities		Charity	
Dining Out		Savings	
Vacation		Car Insurance	
Gas		Phone	

a. Halve the first three fractions in the first column. Since Meghan will have a roommate and will be gone more, she expects to spend half as much on rent, groceries, and utilities. Place the new values in the new table above.

b. Double the last three fractions in the first column. Since Meghan will be traveling, she expects to spend twice as much for dinning out, vacation, and gas. Place the new values in the table.

c. Find the total fractional portion budgeted for the new budget plan if the second column remains the same.

d. Would Meghan be able to afford this new budget on her current income? In other words, do the new fractions in the table add up to a fraction less than 1?

e. If the new budget in part d. is over budget, what fraction is it over? If the new budget is under budget, what fractional portion is left over for miscellaneous items?

Chapter 1 Project

A Trip to the Grocery Store

An activity to demonstrate the use of decimal numbers in real life.

Decimals can be encountered in many areas of our daily lives. When we order food at a restaurant or go to the gas station, we see numbers written and expressed to the nearest hundredth. The grocery store is the same; we see prices for items, prices per unit, sale stickers, and so on, and we need to be able to find the best prices to get the most for our money.

For your upcoming trip to the grocery store, and suppose you have made a full list of items that you will need to purchase. In each question, you will be evaluating the prices of each item on your list using various operations with decimals.

1. The first three items you need to buy are a gallon of milk, 2 cartons of eggs, and a stick of butter. You have found that \$3.49 is the best price for milk, \$2.57 is the best price for one carton of eggs, and \$0.93 is the best price for a stick of butter. What is the total cost of these items?

2. For canned beans, the grocery store is running a promotional deal where you will receive 50 cents off each canned good as long as you buy 10 of them. If the price of one can of beans is \$1.34, what is the total price for 10 cans of beans? Explain how you found your answer.

3. When it comes to buying a loaf of bread, you have several options. To find the best deal, in this case, we need to find the unit price. The unit price is the cost per one item in a larger group. In other words, the unit price of the bread would be the cost per each slice of bread as opposed to the cost for the entire loaf. Assume each slice is the same size.

Bread Brand A has 25 slices and costs \$2.35.
Bread Brand B has 18 slices and costs \$1.95.

a. What is the unit price for a slice of Brand A's bread? Round to the nearest hundredth.

b. What is the unit price for a slice of Brand B's bread? Round to the nearest hundredth.

c. Which brand has the better deal per slice of bread? How do you know?

4. The store has three brands of paper towels for sale, with varying prices and amounts of individual sheets. Assume each sheet has the same dimensions.

Brand C is \$12.90 for 400 sheets.
Brand D is \$8.36 for 500 sheets.
Brand E is \$9.30 for 775 sheets.

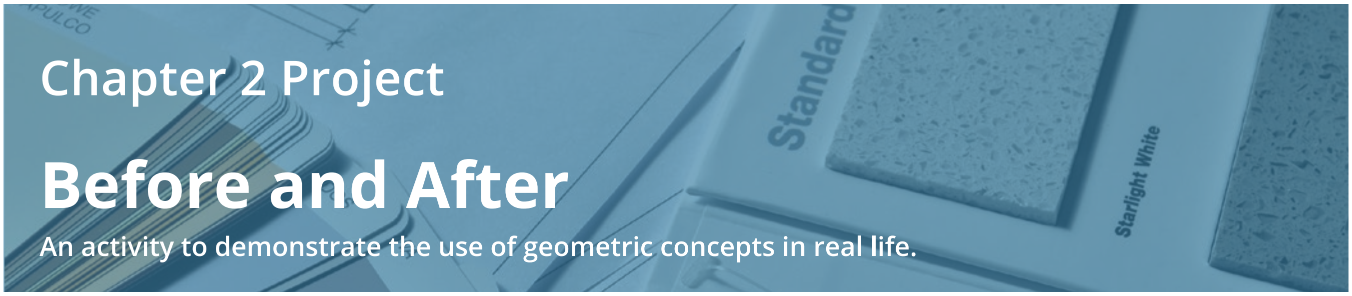
a. Calculate the unit price for each brand per sheet. Round each to the nearest hundredth.

b. During the weekend, the grocery store is running a sale on paper towels. If Brand C is on sale for a discount of 20%, is it now cheaper than Brand E, by unit price? Explain your answer.

c. What total price would Brand D need to be to have the same unit price as Brand E? (Hint: Use the rounded value you found in part a.)

5. To finish your grocery trip, you have to make choices with your remaining money. What is one possible combination of items you could purchase assuming you have \$20.00 left to spend, you must purchase at least one of each item listed, and your total must be greater than \$18.00. Justify your answer by showing your total spent and explain why you chose the items you did.

One pound of bananas for \$0.64
A 5-lb bag of apples for \$5.63
A 3-lb bag of potatoes for \$3.68
A 2-lb bag of baby carrots for \$1.99
A container of guacamole for \$2.67



Chapter 2 Project

Before and After

An activity to demonstrate the use of geometric concepts in real life.

Suppose HGTV came to your home one day and said, "Congratulations, you have just won a FREE makeover for any room in your home! The only catch is that you have to determine the amount of materials needed to do the renovations and keep the budget under \$2000." Could you pass up a deal like that? Would you be able to calculate the amount of flooring and paint needed to remodel the room? Remember it's a FREE makeover if you can!

Let's take an average size room that is rectangular in shape and measures 16 feet 3 inches in width by 18 feet 9 inches in length. The height of the ceiling is 8 feet. The plan is to repaint all the walls and the ceiling and to replace the carpet on the floor with hardwood flooring. You are also going to put crown molding around the top of the walls for a more sophisticated look.

1. Take the length and width measurements that are in feet and inches and convert them to a fractional number of feet and reduce to lowest terms. (Remember that there are 12 inches in a foot. For example, 12 feet 1 inch is $12\frac{1}{12}$ feet.)
2. Now convert these same measurements to decimal numbers.
3. Determine the number of square feet of flooring needed to redo the floor. (Express your answer in terms of a decimal and do not round the number.)
4. If the flooring comes in boxes that contain 24 square feet, how many boxes of flooring will be needed? (Remember that the store only sells whole boxes of flooring.)
5. If the flooring you chose costs \$74.50 per box, how much will the hardwood flooring for the room cost (before sales tax)?
6. Figure out the surface area of the four walls and the ceiling that need to be painted, based on the room's dimensions. (We will ignore any windows, doors, or closets since this is an estimate.)
7. Assume that a gallon of paint covers 350 square feet and you are going to have to paint the walls and the ceilings **twice** to cover the current paint color. Determine how many gallons of paint you need to paint the room. (Again, assume that you can only buy whole gallons of paint. Any leftover paint can be used for touch-ups.)
8. If the paint you have chosen costs \$18.95 per gallon, calculate the cost of the paint (before sales tax).
9. Determine how many feet of crown molding will be needed to go around the top of the room.
10. The molding comes in 12-foot sections only. How many sections will you need to buy?
11. If the molding costs \$2.49 per linear foot, determine the cost of the molding (before sales tax).
12. Calculate the cost of all the materials for the room makeover (before sales tax).
    a. Were you able to stay within budget for the project?
    b. If so, then what extras could you add? If not, what could you adjust in this renovation to stay within budget?
    c. Using sales tax in your area, calculate the final price of the room makeover with sales tax included.

Chapter 2 Project

What's My Average?

An activity to investigate the relationships among different measures of central tendency.

If you are a college student, then grades are important to you. They determine whether you are eligible for scholarships or getting into a particular college or program of choice. It is important to be able to calculate your grade point average in a class and to be able to determine the score you need on a test to reach your desired average. Professors have many different ways of calculating your average for a class. Measures of average are often referred to as measures of central tendency.

For this project, you will be working with two of these measures, the **mean** and the **median**.

Recall that the **mean** of a set of data is found by adding all the numbers in the set and then dividing by the number of data values. The **median** is the middle number once you arrange the data in order from smallest to largest. If there is an even number of data values, then the median is the mean of the two middle values. The median separates the data into two parts such that 50% of the data values are less than the median and 50% are greater than the median.

Jonathan and Tristen are two students in Dr. Hawkes Math 230 class. So far, Dr. Hawkes has given 5 tests and the students' scores are listed below.

Jonathan	Tristen
24	80
98	84
86	88
96	72
96	81

1. Calculate the mean and median of Jonathan's grades.

2. Calculate the mean and median of Tristen's grades.

3. Compare the two measures of *average* for each student.

- **a.** Are the mean and median similar for Jonathan?
- **b.** Are the mean and median similar for Tristen?
- **c.** Based on the **mean**, who has the best *average* in the class?
- **d.** Based on the **median**, who has the best *average* in the class?

4. In your opinion, which student has the most consistent test scores? Explain your reasoning.

5. If each student had scored 2 points higher on each test, how would this affect

- **a.** The mean of their grades?
- **b.** The median?

6. Dr. Hawkes is planning on giving one more test in the class. His grading scale is as follows.

A	93–100
B	85–92
C	74–84
D	69–73
F	Below 69

- **a.** What is the lowest score each student can make on the test and still end up with a grade of C for the class (based on the **mean** of all test scores)?
- **b.** Who has to make the higher grade on the last test to get a C, Jonathan or Tristen?
- **c.** If the last test counts double (equivalent to two test grades) what is the lowest score each student can make on the test in order to make a B in the class (based on the **mean** of all test scores)? (Do not round the mean.)
- **d.** If the last test counts double, who has to make the higher grade on the last test to get a B, Jonathan or Tristen?

7. Based on the work you have done in Problems 1 through 6, which measure do you think is the *best* measure of a student's *average* grade, the mean or the median? (Explain your reasoning by looking at this question from both Jonathan and Tristen's point of view.)

Chapter 3 Project

Going to Extremes!

An activity to demonstrate the use of signed numbers in real life.

When asked what the highest mountain peak in the world is, most people would say Mount Everest. This answer may be correct, depending on what you mean by highest. According to Geology.com, there may be other contenders for this important distinction.

The peak of Mount Everest is 8850 meters (or 29,035 feet) above sea level, giving it the distinction of being the mountain with the highest altitude in the world. However, Mauna Kea is a volcano on the big island of Hawaii whose peak is over 10,000 meters above the nearby ocean floor, which makes it taller than Mount Everest. A third contender for the highest mountain peak is Chimborazo, an inactive volcano in Ecuador. Although Chimborazo only has an altitude of 6263 meters (20,549 feet) above sea level, it is the highest mountain above Earth's center. How could a mountain that is only 6263 meters tall be higher than a mountain that is 8850 meters tall? Because the Earth is not really a sphere but an "oblate spheroid," which means that the Earth is widest at the equator. Chimborazo is 1° south of the equator which makes it about 2000 meters farther from the Earth's center than Mount Everest.

What about the other extreme? What is the lowest point on Earth? As you might have guessed, there is also more than one candidate for that distinction. The lowest exposed area of land on Earth's surface is on the Dead Sea shore at 413 meters below sea level. The Bentley Subglacial Trench in Antarctica is the lowest point on Earth that is not covered by ocean but is covered by ice. This trench reaches 2555 meters below sea level. The deepest point on the ocean floor occurs 10,916 meters below sea level in the Mariana Trench in the Pacific Ocean.

1. Calculate the **difference** in elevation between Mount Everest and Chimborazo in both meters and feet. What operation does the word **difference** imply?

2. Write an expression to calculate the **difference** in elevation between the peak of Mount Everest and the lowest point on the Dead Sea shore in meters and then simplify.

3. If you were to travel from the bottom of the Mariana Trench to the top of Mount Everest, how many meters would you travel?

4. If Mount Everest were magically moved and placed at the bottom of the Mariana Trench, how many meters of water would lie above Mount Everest's peak?

5. How much farther below sea level (in meters) is the Mariana Trench as compared to the Dead Sea shore?

6. Add the elevations (in meters) together for Mount Everest, Chimborazo, the Dead Sea Shore, the Bentley Subglacial Trench, and the Mariana Trench and show your result. Is this number positive or negative? Would this value represent an elevation above or below sea level?

7. Convert the results in Problems 2 through 4 above to feet using the conversion factor 1 meter = 3.28 feet. Do not round your answers.

8. Convert the results in Problem 7 from feet to miles using the conversion factor 1 mile = 5280 feet. (Round your answers to the nearest thousandth.)

9. Using the height of Mount Everest in meters as an example, the conversions in Problems 7 and 8 could have been combined to do the conversion from meters directly to miles by using the following sequence of conversion factors:

$$8850 \text{ m} \cdot \frac{3.28 \cancel{\text{ ft}}}{1 \cancel{\text{ m}}} \cdot \frac{1 \cancel{\text{ m}}}{5280 \cancel{\text{ ft}}} \approx 5.498 \text{ miles}$$

(A mountain peak over 5 miles high!)

Now notice that in doing the conversions, the units for meters and feet cancel out since they appear in both the numerator and denominator, leaving only the unit of miles in the numerator of your result. This is called dimensional analysis and is extremely helpful in converting measurements to make sure you end up with the correct answer and the correct units on your result.

Now verify that this sequence of conversions works by taking the results from Problems 2 through 4 and applying both conversion factors above. How do your results compare to the results from Problem 8? (Round your answers to the nearest thousandth.)

10. There is more than one way that this conversion could have been performed. Using the conversion factors 1 km = 1000 m and 1 mile = 1.61 km, convert the results in Problems 2 through 4 from meters to miles by using these factors in sequence similar to Problem 9 and performing a dimensional analysis. (Round your answer to the nearest thousandth.) Do you get exactly the same results? Why do you think this is so?

Chapter 3 Project

Ordering Operations

An activity to explore the orders of operations.

Did you know that the order of operations you learn in math courses is a relatively new set of rules? The rules were created in the early 1900s and solidified into the current form along with the creation of computers and computer languages. Before the 1600s, mathematical notation was not commonly used, and mathematical expressions and equations were written out in words. Any phrasing that was ambiguous (that is, could be understood in more than one way) was avoided. When simplifying expressions in mathematics, care must be taken to properly follow the order of operations. If you stray from the order, you will reach a different conclusion than intended.

Suppose you are creating a computer program to follow the rules for the order of operations. The computer uses * for multiplication, / for division, and ^ to indicate exponents.

1. As your first test, you enter “8 + 0 * 3 – 10 / 2” into the computer program and it returns a value of 7.

- **a.** Following the order of operations, what would you expect as the result?
- **b.** Explain the error(s) made by the computer program.
- **c.** Insert grouping symbols into the expression so that it will simplify to the value returned by the computer.
- **d.** Rewrite the expression (using parentheses if needed) so that the computer program will return the expected value from part a. if the computer program is not modified.
- **e.** Compare the answers from parts c. and d. What do you notice?

2. After modifying your code, you verify that the previous expression returns the correct value. For the next test, you enter “–4^2 – 10 / 2 + 4” and the computer returns a value of 15.

- **a.** Following the order of operations, what would you expect as the result?
- **b.** Explain the error(s) made by the computer program.
- **c.** Rewrite the original expression so that it will simplify to the value returned by the computer.

3. After further modification, the computer program can now properly follow the order of operations. You next add code to allow the program to simplify algebraic expressions. You test your code by entering “$-2(3x + 5y)$” and it returns $-16xy$.

- **a.** What would you expect the program to return?
- **b.** Explain the error(s) made by the computer program.
- **c.** Is this the same error as the computer returning a result of $30xy$ when you enter “$3(4x + 6y)$”?

4. As a final step, you write code to allow the program to translate English phrases into algebraic expressions. You enter the phrase “five times two plus three” and the program returns “5 * 2 + 3”.

- **a.** If you expected “5(2 + 3)” in return, is the issue with the computer program or your phrasing? Explain what the issue is.
- **b.** Next, you enter the phrase “twelve less two times a number” and the program returns “$2x - 12$”. Is the issue the computer program or phrasing?

Chapter 4 Project

A Linear Vacation

An activity to demonstrate the importance of solving linear equations in real life.

The process of finding ways to use math to solve real-life scenarios is called mathematical modeling. In the following activity you will be using linear equations to model some real-life scenarios that arise during a family vacation.

For each question, write a linear equation in one variable and then solve.

1. Penny and her family went on vacation to Florida and decided to rent a car to do some sightseeing. The cost of the rental car was a fixed price per day plus \$0.29 per mile. When she returned the car, the bill was \$209.80 for three days and they had driven 320 miles. What was the fixed price per day to rent the car?

2. Penny's son Chase wanted to go to the driving range to hit some golf balls. Penny gave the pro-shop clerk \$60 for three buckets of golf balls and received \$7.50 in change. What was the cost of each bucket?

3. Penny's family decided to go to the Splash Park. They purchased two adult tickets and two child tickets. The adult tickets were $1\frac{1}{2}$ times the price of the child tickets and the total cost for all four tickets was \$130. What was the cost of each type of ticket?

4. Penny's family went shopping at a nearby souvenir shop where they decided to buy matching T-shirts. If they bought four T-shirts and a \$9.99 bottle of sunscreen for a total cost of \$89.51, before tax, how much did each T-shirt cost?

5. Penny and her family went out to eat at a local restaurant. Three of them ordered a shrimp basket, but her daughter Meghan ordered a basket of chicken tenders, which was \$4.95 less than the shrimp basket. If the total order before tax was \$46.85, what was the price of a shrimp basket?

6. While on the beach, Penny and her family decided to play a game of volleyball. Penny and her son beat her husband and daughter by two points. If the combined score of both teams was 40, what was the score of the winning team?

Chapter 4 Project

Breaking Even

An activity to demonstrate the use of linear equations and inequalities in business.

In manufacturing, the production cost for an item usually has two components: a fixed cost and a variable cost. You can think of the fixed cost as money that must be spent to operate the business regardless of production level. Examples of fixed cost include paying the rent or mortgage for the manufacturing facility or insurance on the property. The variable cost reflects the funds that must be spent to produce one unit of the product. Variable cost should account for things like raw materials and labor costs.

1. A guitar manufacturer has daily fixed cost of \$12,000 and each guitar costs \$230 to build.
   a. Determine the total cost of producing 10 guitars in one day.
   b. How many guitars were produced in a day when the total cost was \$21,890?
   c. What is the minimum number of guitars produced in a day that will make the total cost exceed \$50,000?
2. The revenue for a manufacturer is the income generated from selling products. Revenue is defined as the price per unit times the number of units sold. The guitar manufacturer from our previous problem can sell each guitar for \$400.
   a. Determine the revenue when 10 guitars are sold.
   b. What is the minimum number of guitars that the manufacturer must sell in a day so that revenue is at least \$50,000?
3. Profit is defined as revenue minus cost. The break-even point is the production level at which the profit is exactly zero. Above that level, the manufacturer returns a profit. Determine the break-even point for this guitar manufacturer. In other words, find the number of guitars that must be manufactured and sold in a day to cover all the manufacturing cost.
4. If the manufacturer could decrease fixed cost from \$12,000 to \$10,000, would you expect the break-even point to go up or down? Explain. Use a linear equation to verify your prediction.
5. Assume that the manufacturer keeps fixed cost at \$12,000 with a production cost of \$230 per guitar. What should the sale price be to keep a break-even point of 80 units?
6. Write a linear inequality that represents the company returning a positive profit if the fixed cost is \$12,000 with a production cost of \$230 per guitar and a sale price of \$400 per guitar.
7. Solve the inequality found in Problem 6. Round to the nearest integer and write the solution set in interval notation. Explain what this solution set represents.
8. Based on the solution set found in Problem 7, is there a limit to the number of guitars that can be manufactured and sold and still return a profit? Do you think this is realistic? Explain your answer.

Chapter 5 Project

What's Your Car Worth?

An activity to demonstrate the use of linear models in real life.

When buying a new car, there are a number of things to keep in mind: your monthly budget, length of the warranty, routine maintenance costs, potential repair costs, cost of insurance, etc.

One thing you may not have considered is the *depreciation*, or reduction in value, of the car over time. If you like to purchase a new car every 3 to 5 years, then the *retention value* of a car, or the portion of the original price remaining, is an important factor to keep in mind. If your new car depreciates in value quickly, you may have to settle for less money if you choose to resell it later or trade it in for a new one.

Below is a table of original Manufacturer's Suggested Retail Price (MSRP) values and the anticipated retention value after 3 years for three 2022 mid-price car models.

Car Model	2022 MSRP	Expected Value in 2025	Rate of Depreciation (slope)	Linear Equation
Mini Cooper	\$28,600	\$16,390		
Toyota Camry	\$25,845	\$13,101		
Ford Taurus	\$30,230	\$13,244		

1. The x-axis of the graph is labeled "Years after Purchase." Recall that the MSRP value for each car is for the year 2022 when the car was purchased.
   a. What value on the x-axis will correspond to the year 2022?
   b. Using the value from part a. as the x-coordinate and the MSRP values in column two as the y-coordinates, plot three points on the graph corresponding to the value of the three cars at time of purchase.
   c. What value on the x-axis will correspond to the year 2025?
   d. Using the value from part c. as the x-coordinate and the expected car values in column three as the y-coordinates, plot three points on the graph corresponding to the value of the three cars in 2025.
2. Draw a line segment on the graph connecting the pair of points for each car model. Label each line segment after the car model it represents and label each point with a coordinate pair, (x, y). Consider using a different color when plotting each line segment to help you identify the three models.

3. Use the slope formula, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, to answer the following questions.

a. Calculate the rate of depreciation for each model by calculating the slope (or rate of change) between each pair of corresponding points using the slope formula and enter it into the appropriate row of column four of the table.

b. Are the slopes calculated above positive or negative? Explain why.

c. Interpret the meaning of the slope for the Toyota Camry making sure to include the units for the variables.

d. Which car model depreciates in value the fastest? Explain how you determined this.

4. Use the slope-intercept form of an equation, $y = mx + b,$ for the following problems.

a. Write an equation to model the depreciation in value over time of each car (in years). Place these in column five of the table.

b. What does the y-intercept represent for each car?

5. Use the equations from Problem 4 for the following problems.

a. Predict the value of the Mini Cooper 4 years after purchase.

b. Predict the value of the Ford Taurus $2\frac{1}{2}$ years after purchase.

6. Determine from the graph how long it takes from the time of purchase until the Ford Taurus and the Toyota Camry have the same value. (It may be difficult to read the coordinates for the point of intersection, but you can get a rough idea of the value from the graph. You can find the exact point of intersection by setting the two equations equal to one another and solving for x.)

a. After how many years are the car values for the Ford Taurus and the Toyota Camry the same? Round to the nearest tenth.

b. What is the approximate value of both cars at this point in time? Round to the nearest 100 dollars.

7. How long will it take for the Toyota Camry to fully depreciate (reach a value of zero)?

a. For the first method, extend the line segment between the two points plotted for the Toyota Camry until it intersects the horizontal axis. The x-intercept is the time at which the value of the car is zero.

b. Substitute 0 for y in the equation you developed for the Toyota Camry and solve for x. Round to the nearest year.

c. Compare the results from parts a. and b. Are the results similar? Why or why not?

8. How long will it take for the Ford Taurus to fully depreciate? (Repeat Problem 7 for the Ford Taurus.) Round to the nearest year.

9. Why is there such a difference in depreciation for the Camry and the Taurus? Do some research on a reliable Internet site and list two reasons why cars depreciate at different rates.

10. Based on what you have learned from this activity, do you think retention value will be a significant factor when you purchase your next car? Why or why not?

Chapter 5 Project

Demand and It Shall Be Supplied

An activity to demonstrate the use of linear equations and linear inequalities in real life.

In economics, the demand for a product is the number of units of the product that the market is willing to absorb at a certain price. That is, the demand is the number of units of the product that sells at any given time. The most basic model for the demand d as a function of the price p is given by a linear equation

$$d = b + mp,$$

where m and b are real numbers.

1. The owner of a T-shirt company believes that the demand d (in units) for their Stranger Things T-shirt follows the linear demand model from the introduction, where p is the price of one T-shirt and m and b are real numbers. The company owner knows that they can sell 300 Stranger Things T-shirts for \$20 each but only 250 T-shirts at \$25 each.
   a. Explain why it is reasonable to believe that the demand of T-shirts decreases as the price increases.
   b. Compute the value of the real number m. How would you interpret the value you have found?
   c. Compute the value of the real number b. How would you interpret the value you have found?
   d. Write the demand equation using your answers from parts b. and c.

The supply for a product is the number of units of a product that the manufacturer can make available at a certain price. The most basic model for the supply s as a function of the price p is given by a linear equation

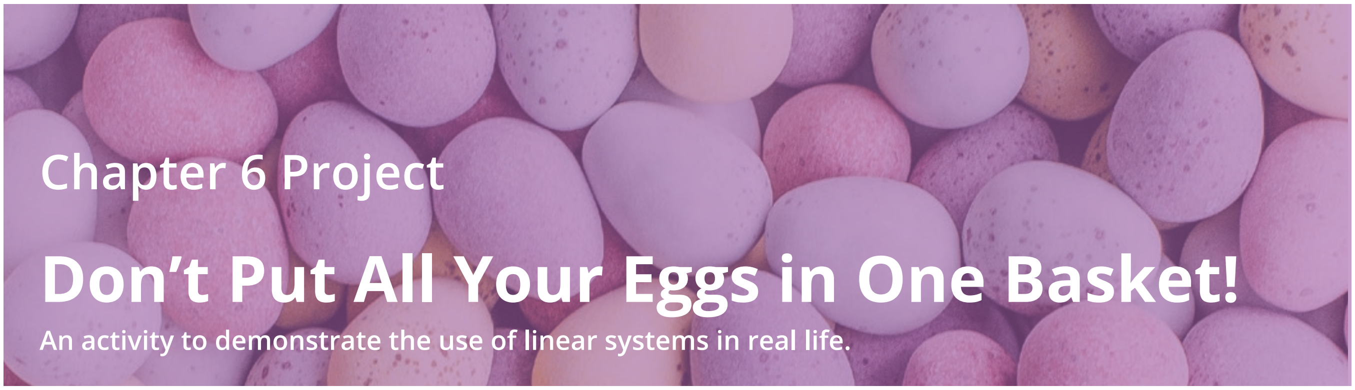
$$s = b + mp,$$

where m and b are real numbers.

2. The T-shirt company from Problem 1 can produce 275 shirts when the price is \$20. If the price is raised to \$25, they can produce 300 shirts.
   a. Explain why it is reasonable to believe that the supply of T-shirts increases as the price increases.
   b. Compute the value of the real number m. How would you interpret the value you have found?
   c. Compute the value of the real number b. How would you interpret the value you have found?
   d. Write the supply equation using your answers from parts b. and c.

3. The equilibrium price is the price for which the demand is equal to the supply. At this price, the number of units produced is exactly the number of units absorbed by the market.
   a. Graph the demand and supply equations for the T-shirt company on the same coordinate plane. Recall that the x-axis should represent price.
   b. Find the equilibrium price rounded to the nearest cent and explain what it means in words.

4. Some products have a demand that is not sensitive to changes in price. That is, a large variation in price will not produce a corresponding large variation in demand. This phenomenon is defined as inelastic demand. Perform an internet search to find an example of a product with an inelastic demand. Explain why the demand for the product is inelastic.



Chapter 6 Project

Don't Put All Your Eggs in One Basket!

An activity to demonstrate the use of linear systems in real life.

Have you ever heard the phrase "Don't put all your eggs in one basket"? This is a common saying that is often quoted in the investment world—and it's true. In an ever-changing economy it is important to diversify your investments. Splitting your money up into two or more funds may keep you from losing it all if one of the funds performs poorly. You may be thinking that you are too young to consider investments and saving money for retirement, but it is never too soon—especially in an economy where interest rates are extremely low. Low rates means it takes even longer to build up your nest egg. So start saving now and be sure to have more than one basket to put your eggs in! For this activity, if you need help understanding some of the investment terms, use the following website as a resource:
www.investopedia.com

Let's suppose that you received a total of $5000 in cash as a graduation present from your relatives. You also have an additional $2500 that you saved from your summer job. You are thinking about investing the $7500 in two investment funds that have been recommended to you. One is currently earning 4% interest annually (conservative fund) and the other is earning 8% annually (aggressive fund). Keep in mind that interest rates fluctuate as the economy changes and there are few guarantees on the amount you will actually earn from any investment. Also, note that higher rates of interest typically indicate a higher risk on your investment.

1. If you want to earn $400 total in interest on your investments this year, how much money would you have to invest in each fund? Let the variable x be the amount invested in Fund 1 and the variable y be the amount invested in Fund 2. Recall that to calculate the interest on an investment, use the formula $I = Prt$, where P is the principal or amount invested, r is the annual interest rate, and t is the amount of time invested, which for our problem will be 1 year ($t = 1$). Use the table below to help you organize the information. Note that interest rates have to be converted to decimals before using them in an equation.

	Principal	**Interest Rate**	**Interest**
Fund 1	x	0.04	$0.04x$
Fund 2	y	0.08	$0.08y$
Total	**a.**		**b.**

a. Fill in the total amount available for investment in the bottom row of the table.

b. Fill in the total amount of interest desired in the bottom row of the table.

c. What does $0.04x$ represent in the context of this problem?

d. What does $0.08y$ represent in the context of this problem?

e. Using the principal column of the table, write an equation in standard form involving the variables x and y to represent the total amount available for investment.

f. Using the interest column of the table, write an equation in standard form involving the variables x and y to represent the total amount of interest desired.

g. Solve the linear system of two equations derived in parts e. and f. to determine the amount to invest in each fund to earn $400 in interest. (You may use any method you choose: substitution, addition/elimination, or graphing).

h. Check to make sure that your solution to the system is correct by substituting the values from part g. for x and y into both equations and verify that the equations are true statements.

2. Suppose you decide you want to earn more interest on your investment. You now want to earn \$500 in interest next year instead of \$400. Using a table similar to the one in Problem 1, organize the information and follow a similar format to determine the amounts to invest in each of the funds that will earn \$500 in interest in a year.

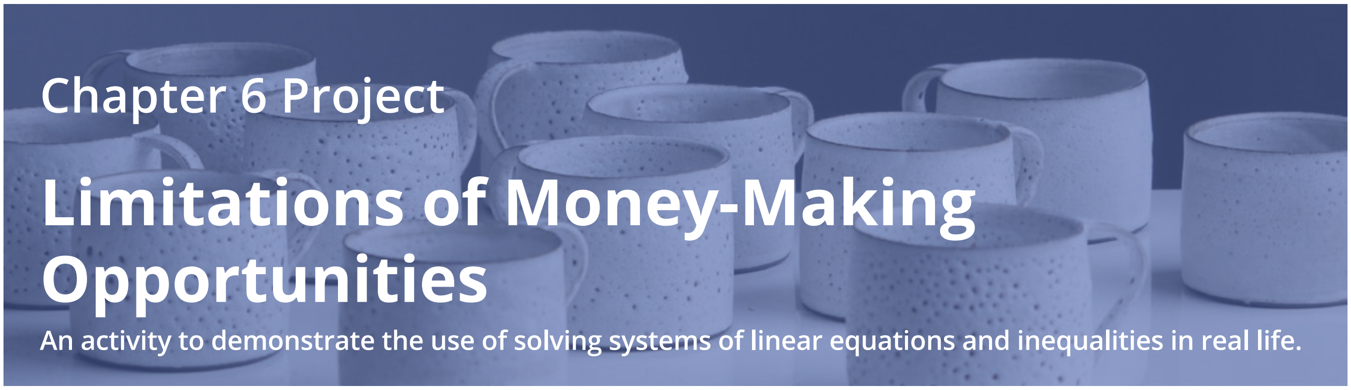
3. Compare the results you obtained from Problems 1 and 2. How did the amounts in each investment change when your desired interest increased by \$100?

4. Suppose you decide that \$500 is not enough interest and you want to earn an additional \$100 on your investments for a total of \$600 in interest. Using a table similar to the one in Problem 1, organize the information and follow a similar format to determine the amounts to invest in each of the funds that will earn \$600 in interest in a year.

5. Compare the results from Problem 4 to the results from Problems 1 and 2.
   - **a.** How much are you investing in Fund 1 to earn \$600 in interest?
   - **b.** How much are you investing in Fund 2 to earn \$600 in interest?
   - **c.** How do your results contradict the advice provided to you at the start of this activity?
   - **d.** Is it possible to make more than \$600 in interest on your \$7500 investment using these two funds? Explain why or why not?
   - **e.** What is the smallest amount of interest you can earn on your investment using these two funds? How did you determine this?

6. How much interest would you earn if you split the initial principal of \$7500 equally between the two funds?

7. If you actually had \$7500 to invest in these two funds earning 4% and 8% respectively, how would you invest the money? Explain your reasoning.



Chapter 6 Project

Limitations of Money-Making Opportunities

An activity to demonstrate the use of solving systems of linear equations and inequalities in real life.

In this project, you will investigate how limitations (known as *constraints*) on the production of goods can lead to limitations on how much profit a company can make.

Suppose the Soaring Eagle Book Store wishes to produce two types of coffee mugs: Type A and Type B. Each Type A mug will result in a profit of \$3, and each Type B mug will result in a profit of \$2.75. Manufacturing one Type A mug requires 6 minutes on Machine I and 3 minutes on Machine II. Manufacturing one Type B mug requires 4 minutes on Machine I and 6 minutes on Machine II. According to the schedule, Machine I has 12 hours available and Machine II has 10 hours available to make the mugs. How many of each type of mug should the bookstore make to maximize its profit?

1. Using the variables x and y, identify the two unknown quantities.

2. Consider the information provided about the time available on each machine. This information allows us to write two linear inequalities in the standard form $Ax + By \leq C$. These are called constraints.

   a. Explain what each constraint represents.

   b. Explain why the constraints should be of the less-than-or-equal-to type.

   c. Write the two constraints. (**Note:** Be mindful of how the times are expressed, minutes versus hours. You'll need to convert to make units consistent throughout.)

3. Regardless of maximizing profit, there are two more "common sense" constraints. Consider whether there is a minimum or maximum number of mugs. Using this information, write two more constraints and explain why they are constraints.

4. Graph the system of linear inequalities resulting from the four constraints. (**Hint:** You may wish to first graph the two inequalities from Problem 3, shading lightly. Once you've identified the intersection, erase extra shading. Then add the other two constraints found in Problem 2 to the graph and, once you've identified the new (smaller) intersection, erase extra shading again.)

5. The final result of graphing the four constraints should be a four-cornered region. Use systems of equations to solve for the (x, y) coordinates of the four corners.

6. Recall the profit that the bookstore can make from each type of mug. Write an equation of the form $P = Ax + By$, with A and B filled in but not P.

7. Consider the following six values of P: -400, -200, 0, 200, 400, and 600. Using your answer to Problem 6, lightly graph six lines for these six different values of P over the shaded four-cornered region from Problem 5. (**Hint:** These six lines will be parallel.)

8. Would all of the lines created in Problem 7 indicate a possible value of P? Explain why or why not.

9. Is there a maximum value of P or can it keep growing? Why or why not?

10. Let's determine the maximum profit that Soaring Eagle Book Store can make from selling mugs.

   a. If there is a maximum value of P, at what point does it seem to occur?

   b. Is there anything special about the point found in part a.?

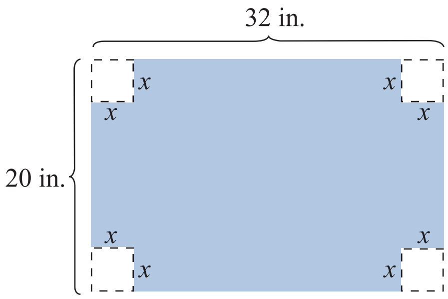
   c. Substitute this point into your $P = Ax + By$ equation from Problem 6 to calculate the maximum profit. Express your final answer in words; that is, state how many mugs of each type should be made to produce the maximum profit and what that maximum profit is.

Chapter 7 Project

Math in a Box

An activity to demonstrate the use of polynomials in real life.

Suppose you have a piece of cardboard with a length of 32 inches and a width of 20 inches and you want to use it to create a box. You would need to cut a square out of each corner of the cardboard so that you can fold the edges up. But what size squares should you cut? Cutting four small squares will make a shorter box. Cutting four large squares will make a taller box.



1. Since we haven't determined the size of the square to cut from each corner, let the side length of the square be represented by the variable x. Write a simplified polynomial expression in x and note the degree of the polynomial for each of the following geometric concepts.
   - **a.** The length of the base of the box once the corners are cut out
   - **b.** The width of the base of the box once the corners are cut out
   - **c.** The height of the box
   - **d.** The perimeter of the base of the box
   - **e.** The area of the base of the box
   - **f.** The volume of the box

2. Evaluate the volume expression for the following values of x. (Be sure to include the units of measurement.)
   - **a.** $x = 1$ in.
   - **b.** $x = 2$ in.
   - **c.** $x = 3$ in.
   - **d.** $x = 3.5$ in.
   - **e.** $x = 6$ in.
   - **f.** $x = 7$ in.

3. Based on your volume calculations for the different values of x in Problem 2, if you were trying to maximize the volume of the box, between what two values of x do you think the maximum will be?

4. Using trial and error, see if you can determine the side length x of the square that maximizes the volume of the box. (**Hint:** It will be a value in the interval from Problem 3.)

5. Using the value you found for x in Problem 4, determine the dimensions of the box that maximize its volume.

6. Calculate the volume of the box in Problem 5.

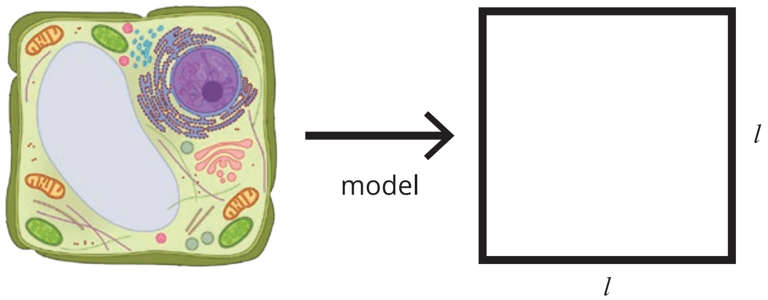
Chapter 7 Project

Small but Mighty

An activity to investigate the use of polynomials in biology.

In biology, we define a cell as the basic unit that contains the essential molecules of life and that all living things are composed of. In this activity, we will investigate why cells are usually very small.

Using a very simplified model, we can consider a cell as a small square. The sides of the square model the cell's membrane, which is the way the cell interacts with the environment, and the inside of the square models the cell's cytoplasm and nucleus, which are the parts that make the cell function.



1. What is the perimeter of a cell whose side length is equal to 10 μm (micrometers)?
2. What is the area of this same cell?
3. Compute the perimeter and the area for cells whose side lengths are 20 μm, 40 μm, and 80 μm. Summarize your findings in the table below.

Side Length (μm)	Perimeter (μm)	Area (μm^2)
10		
20		
40		
80		

4. If a cell has a side length given by l, write a polynomial that represents its perimeter using the variable l. What is the degree of the polynomial you found?
5. If a cell has a side length given by l, write a polynomial that represents its area using the variable l. What is the degree of the polynomial you found?

We know that the ability of a cell to obtain its nutrients is proportional to its perimeter. That is, when a cell doubles the length of its side, it will be able to obtain twice as many nutrients. We also know that the amount of nutrients needed by a cell to function is proportional to its area. If a cell doubles in area, its need for nutrients also doubles.

6. Looking at the table you completed in Problem 3, determine how many times the need for nutrients increases every time the length of the side of the cell doubled.
7. Use your answer from Problem 6 to explain why there might be a limit to a cell's size. Explain why the need for nutrients grows much faster than the growth in the ability to obtain these nutrients.
8. Do a quick internet search and find out what the smallest and largest cells found in nature are. Does the answer surprise you? Why?

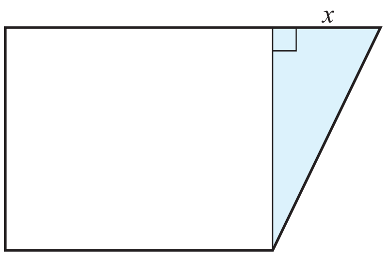
Chapter 8 Project

Building a Dog Pen

An activity to demonstrate the use of the Pythagorean Theorem and quadratic equations in real life.

Justin's house sits on a lot that is shaped like a trapezoid. He decides to make the back part of the lot usable by building a triangular dog pen for his dog Blackjack. On his lunch break at work, he decides to order the materials but realizes that he forgot to write down the actual dimensions of that area of the lot. He wants to get the pen done this weekend because his friends are coming over to help him, so he has to order the materials today so they arrive on time. Justin remembers that one of the sides is 5 feet more than the length of the shortest side and the longest side is 10 feet more than the shortest side. Can he use this information to figure out the dimensions of the area so that he can order the materials today?

1. Using the diagram of the lot provided and the variable x for the length of the shortest side of the dog pen, write an expression for the other two sides of the triangular pen and label them on the diagram.



2. Using the Pythagorean Theorem, substitute the three expressions for the sides of the triangle into the formula and simplify the resulting polynomial. Be sure to move all terms to one side of the equation with the other side equal to zero. (Remember that the longest side is the hypotenuse in the formula. Make sure your leading term has a positive coefficient and that your squared binomials result in a trinomial.)

3. Use the equation you found in Problem 2 to answer the following.

a. Factor the resulting quadratic equation into two linear binomial factors.

b. Find the two solutions to the equation from part a. using the zero-factor law.

c. Do both of these solutions make sense? Explain your reasoning.

d. Using the solution (or solutions) that makes sense, substitute this value for x and determine the dimensions of the dog pen.

4. To fence in the dog pen, Justin plans to purchase chain-link fencing at a cost of $2.58 per foot.

a. How much fencing will he need?

b. How much will the fencing cost?

5. How much area will the dog pen enclose?

6. Justin decides to also put a dog house in the pen to protect Blackjack in bad weather. The dog house is rectangular in shape and measures 2.5 feet by 3 feet. Once the dog house is in the pen, how much area will Blackjack have to run in?

7. The sides of the pen form a right triangle, and the measurements of the sides of the pen were found using the Pythagorean Theorem. Any three positive integers that satisfy the Pythagorean Theorem are called a **Pythagorean triple**. There are an infinite number of these triples and numerous formulas that can be used to generate them. Do some research on the internet to find one of these formulas and use the formula to generate three more sets of Pythagorean triples. Verify that they are Pythagorean triples by substituting them into the Pythagorean Theorem.

8. Another way to generate a Pythagorean triple is to take an existing triple and multiply each integer by a constant. Take the Pythagorean triple $(3, 4, 5)$ and multiply each integer in the triple by the factors below and verify that the result is also a Pythagorean triple by substituting into the Pythagorean Theorem.

a. Multiply by 3:

b. Multiply by 5:

c. Multiply by 8:

Chapter 8 Project

Planting Seeds

An activity to demonstrate factoring polynomials in real life.

Imagine you live within walking distance from a community garden where anyone in the neighborhood can plant, grow, and harvest various vegetables and plants. You have been given a plot of land in this garden that is in the shape of a rectangle. You also have been given seeds to plant tomatoes, peppers, cucumbers, and squash.

When planting seeds in a garden, it is important to know how much space you have for the crops, and to adjust your seeds accordingly. One possible way the plot of land could be divided is shown. For Problems 1 through 5, assume this is the layout of the vegetables.

Tomatoes	Cucumbers	Squash	Tomatoes
Tomatoes	Cucumbers	Squash	Tomatoes
Peppers	Cucumbers	Squash	Peppers

Width

Length

1. The squash sections each have a width of $2x - 1$ feet. What is the total width of the column of squash?

2. If the length of the column of cucumbers is $x - 4$, and the width of each section of cucumbers is $4x + 1$, what is the total area of the three cucumber sections?

3. The section of tomatoes on the left side of the plot of land has an area of $x^2 + 6x + 9$ ft^2. If this plot of tomatoes were in the shape of a square, what would be the length of its side?

4. One squash section has a width of $2x - 1$ feet and a length of $4x + 2$ feet.

  a. What is the area of this section of squash?

  b. Is there any other possible width and length of this plot of land that can exist with this given area? Explain why or why not.

5. The area of one section of peppers is $x^2 + bx + 24$ ft^2, where b is an unknown positive value.

  a. What are several different options for the value of b so that this area can be factored? Explain how you arrived at your answer(s).

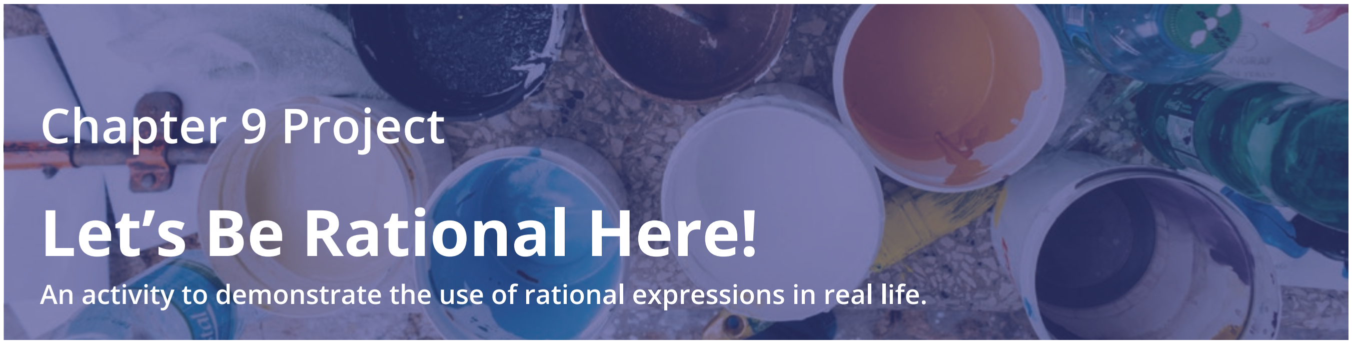
  b. If the area of the section of peppers were $x^2 + bx - 24$ ft^2 instead, how would this change your answers to part a.?

For Problems 6 through 8, assume you are looking at a plot of land next to yours.

6. The total area of a square plot of land next to yours is $4x^2 + 20x + 25$. What is the length of one side of the square? Explain how you arrived at your answer.

7. The area of one section of tomatoes on the right side of the plot of land is $3x^2 - 2x - 5$ ft^2. What are the dimensions of this section? Explain how you arrived at your answer.

8. Assume you wanted to plant the cucumbers in the shape of a square. Is it possible for the area of a cucumber section to be $x^2 + 9$ ft^2 and have one side length of $x + 3$? Explain why or why not.



Chapter 9 Project

Let's Be Rational Here!

An activity to demonstrate the use of rational expressions in real life.

You may be surprised to learn how many different situations in real life involve working with rational expressions and rational equations. Hopefully after spending the day with Meghan and her friends you'll be convinced of their importance.

It's Saturday and Meghan has a list of things to accomplish today: revise her budget, paint the walls in the spare bedroom, travel to the lake with her friends and then water ski on the lake for the rest of the day, provided the weather stays nice.

1. Meghan decides to tackle the budget first. After reviewing her budget, she decides that she needs to get a part-time job to earn some extra money. She is remodeling the living room and would like to buy some new furniture. Letting x represent her new monthly combined salary, she estimates that $\frac{1}{4}$ of her new salary will be used for bills and approximately $\frac{1}{5}$ for her car payment. She would like to have \$1100 left over each month, of which \$100 will be saved for the new furniture. What must her new monthly salary be?

- **a.** Let the variable x represent Meghan's new monthly salary. Write an expression to represent the amount of her new salary used for bills. (Remember that the word **of** implies multiplication.)
- **b.** Write an expression to represent the amount of Meghan's new monthly salary used for her car payment.
- **c.** Write an equation that sums the expenditures and leftover balance, and set this sum equal to the new monthly salary of x.
- **d.** Find the LCD of the rational expressions from part c. and multiply it by each term in the equation to remove the fractions.
- **e.** Solve the equation from part d. to determine what Meghan's new monthly salary needs to be.
- **f.** Approximately how much of Meghan's new salary will be used to pay her car payment?

2. With the budget done, Meghan prepares to paint the spare bedroom. She painted her bedroom last week and it took her about 4 hours. Her housemate Ashley painted her bedroom a couple of weeks ago and it took her 6 hours. All the bedrooms are similar in size. Meghan realizes that if she gets Ashley to help her with the painting, it will take them less time and they can get to the lake sooner. How long will it take Meghan and Ashley working together to paint the spare bedroom? Use the table below to help you set up the problem.

Person	Time (in hours)	Part of Work Done in 1 Hour
Meghan	4	$\frac{}{}$
Ashley	6	$\frac{}{}$
Together	x	$\frac{1}{x}$

a. Fill in the missing information in column three of the table. Use the entries in the last column of the table to set up an equation to represent the sum of the amount of work done by both Meghan and Ashley in an hour.

b. Solve the equation to determine how long it will take the two girls to paint the spare bedroom when working together. Express the result in hours as a decimal rounded to the nearest tenth. Convert this measurement to hours and minutes.

3. After the spare bedroom is painted, the girls call Lucas to let him know they are ready to head to the lake. While he is preparing the boat for the lake, Lucas tries to decide which route they should travel. If he travels the highway, he can travel 20 mph faster than the scenic route. However, the highway is 30 miles longer than the scenic route, which is 60 miles long. Lucas thinks it should take him the same amount of time to get there using either route. Use the following table to help you organize the information for this problem.

	Distance (miles)	÷	Rate (mph)	=	Time (hours)
Highway	90		$x + 20$		______
Scenic Route	60		x		____

a. Fill in the missing information in column four of the table. (**Hint:** Recall that the formula that involves distance, rate, and time is $d = r \cdot t$.)

b. Since Lucas expects the time to be the same for each route, create an equation from the last column and solve for x.

c. How fast must Lucas travel on the highway to get to the lake in the same amount of time as traveling the scenic route?

Chapter 9 Project

You Are What You Eat

An activity to demonstrate the use of rational expressions in real life.

When you eat something, your body immediately reacts and responds to the food in a variety of different ways. For example, signals are sent to your brain from your stomach to help indicate when you're full and don't need any more food. Similarly, the central nervous system will calm your body down to indicate a state of relaxation and safety while eating. Amongst thousands of tiny operations your body does while eating, one such action your body takes is regulating the state of acidity inside the mouth based on the food that enters it. We are going to look at two common states our body experiences: eating food and taking an aspirin. Acidity is measured using a scale called pH.

This pH in a human can be determined by the formula $\text{pH} = \frac{20.4x}{x^2+36} + 6.5$, where x is the number of minutes that have passed since the food has been eaten. Normally, the body has a pH of about 7.35 to 7.45 on a scale of 0 to 14.

1. Simplify the equation so that the pH equation is expressed as a single rational expression.

2. Using your simplified equation, determine the acid level after 25 minutes. Round to the nearest hundredth.

3. Assume the pH in a human can be determined by the formula $\text{pH} = \frac{20.4x}{x^2+36} + 6.5$, but the formula for the pH in a rat can be found by the formula $\text{pH} = \frac{0.3\left(x^2-2x\right)}{x^3+x^2-6x} + 6.5$.
   a. Simplify the equation for rat pH so that the equation is expressed as a single rational expression.
   b. Divide the pH formula for a rat by the pH formula for a human. Write the resulting expression as a product of two polynomials in the numerator and the denominator.

4. Does it take longer for the pH level of a rat or a human to level out and normalize? (**Hint:** Try graphing the equations using a graphing calculator or desmos.com/calculator.)
   a. Explain how you arrived at your answer.
   b. Hypothesize what this conclusion means for the pH level of a human and the pH level of a rat.

When an aspirin is taken, there is a concentration of the medication that can be found in the blood. The blood concentration of aspirin brand A is represented by $f(x) = \frac{2x}{3x^2-4x+5}$ where x is in hours. The concentration of aspirin brand B is represented by $g(x) = \frac{x}{2x^2-4x+10}$ where x is in hours.

5. Add the two functions together to find the new function that represents both aspirins in the bloodstream simultaneously. What is the new function? Make sure to distribute and simplify your answer as much as possible.

6. Which of the following three values is greatest? Explain your answer and show your work for each calculation. Round each value to the nearest hundredth.
   a. The concentration of aspirin brand A in the blood after 3 hours
   b. The concentration of aspirin brand B in the blood after 3 hours
   c. The concentration of both aspirin in the blood after 6 hours

Chapter 10 Project

Let's Get Radical!

An activity to demonstrate the use of radical expressions in real life.

There are many different situations in real life that require working with radicals, such as solving right-triangle problems, working with the laws of physics, calculating volumes, and solving investment problems. Let's take a look at a simple investment problem to see how radicals are involved.

The formula for computing compound interest for a principal P that is invested at an annual rate r and compounded annually is given by $A = P(1+r)^n$, where A is the accumulated amount in the account after n years.

1. Let's suppose that you have \$5000 to invest for a term of 2 years. If you want to make \$600 in interest, then at what interest rate should you invest the money?

a. One way to approach this problem would be through trial and error, substituting various rates for r in the formula. This approach might take a while. Using the table below to organize your work, try substituting 3 values for r. Remember that rates are percentages and need to be converted to decimals before using the formula. Did you get close to \$5600 for the accumulated amount in the account after 2 years?

Annual Rate (r)	Principal (P)	Number of Years (n)	Amount, $A = P(1+r)^n$
	\$5000	2	
	\$5000	2	
	\$5000	2	

b. Let's try a different approach. Substitute the value of 2 for n and solve this formula for r. Verify that you get the following result: $r = \sqrt{\frac{A}{P}} - 1$ (**Hint:** First solve for $(1+r)^2$ and then take the square root of both sides of the equation.) Notice that you now have a radical expression to work with. Substitute \$5000 for P and \$5600 for A (which is the principal plus \$600 in interest) to see what your rate must be. Round your answer to the nearest percent.

2. Now, let's suppose that you won't need the money for 3 years.

a. Use $n = 3$ years and solve the compound interest formula for r.

b. What interest rate will you need to invest the principal of \$5000 at in order to have at least \$5600 at the end of 3 years? (To evaluate a cube root you may have to use the rational exponent of $\frac{1}{3}$ on your calculator.) Round to the nearest percent.

c. Compare the rates needed to earn at least \$600 when $n = 2$ years and $n = 3$ years. What did you learn from this comparison? Write a complete sentence.

3. Using the above formulas for compound interest when $n = 2$ years and $n = 3$ years, write the general formula for r for any value of n.

4. Using the formula from Problem 3, compute the interest rate needed to earn at least \$3000 in interest on a \$5000 investment in 7 years. Round to the nearest percent.

5. Do an internet search on a local bank or financial institution to determine if the interest rate from Problem 4 is reasonable in the current economy. Using three to five sentences, briefly explain why or why not.

Chapter 10 Project

Rationally Increasing Precision in Population Problems

An activity to demonstrate the use of rational exponents in real life.

Sometimes we need to find the value of an exponential expression where the exponent is not an integer. This often happens when dealing with exponential growth. In the essay "Observations Concerning the Increase of Mankind, Peopling of Countries, etc.," written in 1751, Benjamin Franklin projected that the human population in the thirteen US colonies was doubling in size every twenty–five years. For example, if one year the population was 300,000 people, then to estimate the number of people 75 years later, calculate $300{,}000 \cdot 2^3 = 2{,}400{,}000$. Note that this works because $\frac{75}{25} = 3$, meaning the population would double 3 times in 75 years. Also notice that since $2 \cdot 2 \cdot 2 = 2^3 = 8$, we multiply 300,000 by 8 to get the final population.

If the timespan we want to estimate the future population for is not a multiple of 25, we can still calculate this value using rational exponents (with as much precision as we like). This investigation will suggest which types of numbers can be exponents, a topic which will be expanded upon in future math courses.

In the following investigation, do not round the exponents. Round the answers to have 10 digits, if necessary.

1. Calculate 2 to each of the following powers.

- **a.** 3
- **b.** 3.1
- **c.** 3.14
- **d.** 3.141
- **e.** 3.1415
- **f.** 3.14159
- **g.** 3.141592
- **h.** 3.1415926
- **i.** 3.14159265

2. The sequence of exponents in Problem 1 is approaching which special number? (**Hint:** See Section 2.1.)

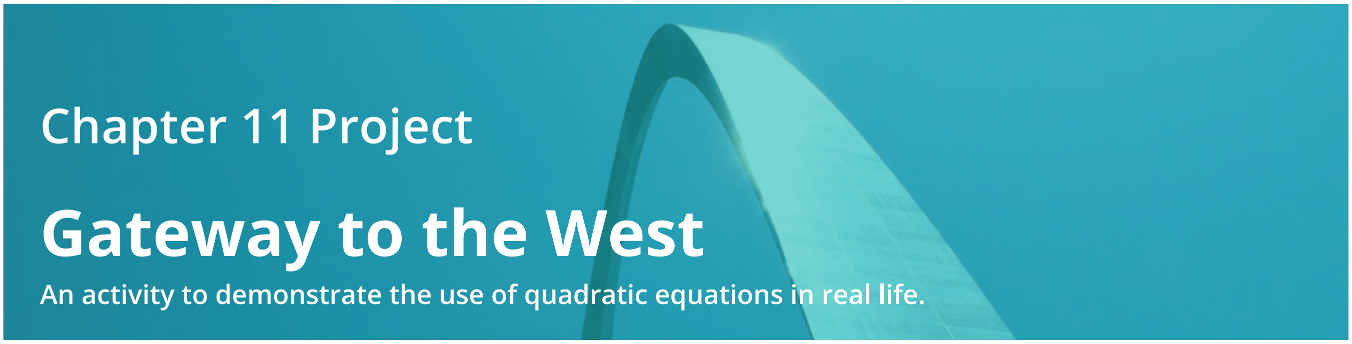
3. In Problem 1, are the calculated powers of 2 increasing, decreasing, or is there no discernible pattern? If increasing (or decreasing), are they increasing (or decreasing) toward a particular value?

4. Consider raising the value 2 to the power of the special number found in Problem 2.

- **a.** If it is possible, state the result and compare it to the results of Problem 1. If this is not possible, explain why.
- **b.** How does this relate to your answer in Problem 3? If there is no relation, explain why.

5. Returning to Franklin's population prediction, consider if someone wanted to know the population 77.5 years later, instead of 75 years later.

- **a.** Determine the decimal form of the exponent $x = \frac{77.5}{25}$.
- **b.** Substitute the value of x found in part a. into the population equation and simplify: $300{,}000 \cdot 2^x$.
- **c.** Explain what x stands for. Interpret the answer to part b. In this case, does it make sense to round or not? If rounding does make sense, what place would you round to and what is the result?
- **d.** If the population starts at 300,000, how many years have passed if $300{,}000 \cdot 2^{3.14}$ provides an estimate of the population size?



Chapter 11 Project

Gateway to the West

An activity to demonstrate the use of quadratic equations in real life.

The Gateway Arch on the St. Louis riverfront in Missouri serves as an iconic monument symbolizing the westward expansion of American pioneers, such as Lewis and Clark. A nationwide competition was held to choose an architect to design the monument and the winner was Eero Saarinen, a Finnish American who immigrated to the United States with his parents when he was 13 years old. Construction began in 1962, and the monument was completed in 1965. The Gateway Arch is the tallest monument in the United States. It is constructed of stainless steel and weighs more than 43,000 tons. Although the arch is heavy, it was built to sway with the wind to prevent it from being damaged. In a 20 mph wind, the arch can move up to 1 inch. In a 150 mph wind, the arch can move up to 18 inches.

1. If you were to place the Gateway Arch on a coordinate plane centered around the y-axis, then the equation $y = -0.00635x^2 + 630$ could be used to model the height of the arch in feet.

a. The general form for a quadratic function is $y = ax^2 + bx + c$. Identify the values for a, b, and c from the Gateway Arch equation.

b. Find the vertex of the Gateway Arch equation.

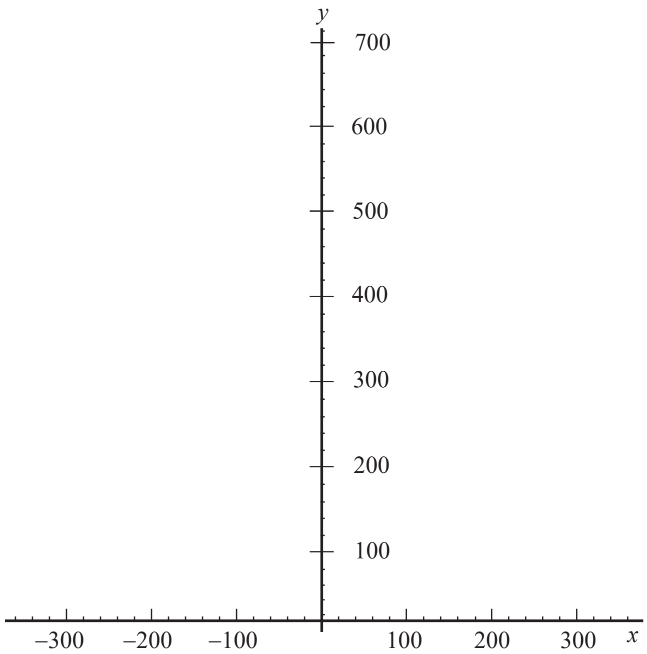
c. Does the vertex represent a maximum or a minimum? Explain your answer based on the coefficients of the Gateway Arch equation.

d. What is the height of the Gateway Arch at its peak?

e. Write the equation for the axis of symmetry of the Gateway Arch equation.

f. Find the x-intercepts of the Gateway Arch equation. Round to the nearest integer.

2. Using the coordinate plane below and the information from Problem 1, graph the Gateway Arch equation.



3. How far apart are the legs of the Gateway Arch at its base?

4. The Gateway Arch equation is a mathematical model. Look up the actual values for the height of the Gateway Arch and the distance between the legs of the arch at its base on the internet and describe how they compare to the values calculated using the equation.

Chapter 11 Project

Determining Product Pricing to Maximize Revenue

An activity to demonstrate the use of quadratic equations in real life.

A company determines that if p is the price charged for an electric bicycle they manufacture, then the number of bikes that will sell n is a function of p: $n(p) = 3750 - 3p$. This is because for each $1 increase in the price, three fewer bikes are sold. The revenue R earned is also a function of p because it is the number of bikes sold times the price per bike: $R(p) = p(3750 - 3p) = -3p^2 + 3750p$. Using methods learned in this chapter, we will investigate just how much revenue the bike manufacturer can earn. Is it boundless or is there a maximum?

For simplicity, in order to compare variable names used within the chapter, let's replace p with x and R with y, so $y = x(3750 - 3x)$. Notice that the graph of this equation is a parabola.

1. Solve the equation $x(3750 - 3x) = 0$ for x. Notice that by setting $y = 0$, you are solving for the x-coordinates of the two x-intercepts. State the coordinates of the two x-intercepts.

2. Use the equation $y = x(3750 - 3x)$ for the following.

  a. Choose an x-value smaller than the smallest x-value of the x-intercepts found in Problem 1 and calculate y.

  b. Choose an x-value larger than the largest x-value of the x-intercepts found in Problem 1 and calculate y.

  c. What do you notice about y-values found in parts a. and b? Recall that y is revenue and x is price. Can you conclude an initial interval for price that the company should stay within?

3. Calculate the mean of the two x-values found in Problem 1.

4. Pick two new values of x between the smallest x-value from Problem 1 and the answer to Problem 3. Next, pick two values of x strictly between the answer to Problem 3 and the largest x-value from Problem 1.

5. Arrange these four values from least to greatest and include your answer from Problem 3, for a total of five unique values.

6. **a.** Evaluate $y = x(3750 - 3x)$ for these five values of x from Problem 5. This will give you five points on the parabola.

  b. For the smallest of the x-values from part a, explain in words what the (x, y) coordinates represent. Include the values.

7. Carefully choose a horizontal and vertical scale and plot the five points found in Problem 6 part a. Also plot the x-intercepts found in Problem 1. Use these points to sketch the parabola.

8. Based on your values and the graph, state the (x, y) coordinates of the vertex (or your best approximation).

9. Is the y-coordinate of the vertex a minimum or maximum value of y? Why do you think this is, based both on the context and on the function?

10. Use the vertex formula and the equation $y = x(3750 - 3x) = -3x^2 + 3750x$ to find the vertex of the parabola.

11. Does the vertex found in Problem 10 support your approximation of the vertex found in Problem 8?

12. Now return to the question about revenue from selling electric bicycles.

  a. What is the lower bound on how much revenue the company can earn?

  b. What is the upper bound on how much revenue the company can earn? What price is charged for that to be the revenue earned?

  c. How many bikes need to be sold to reach the maximum revenue found in part b? (**Hint:** Recall the number of bikes sold is given by $n(p) = 3750 - 3p$ where p is the price per bike.)