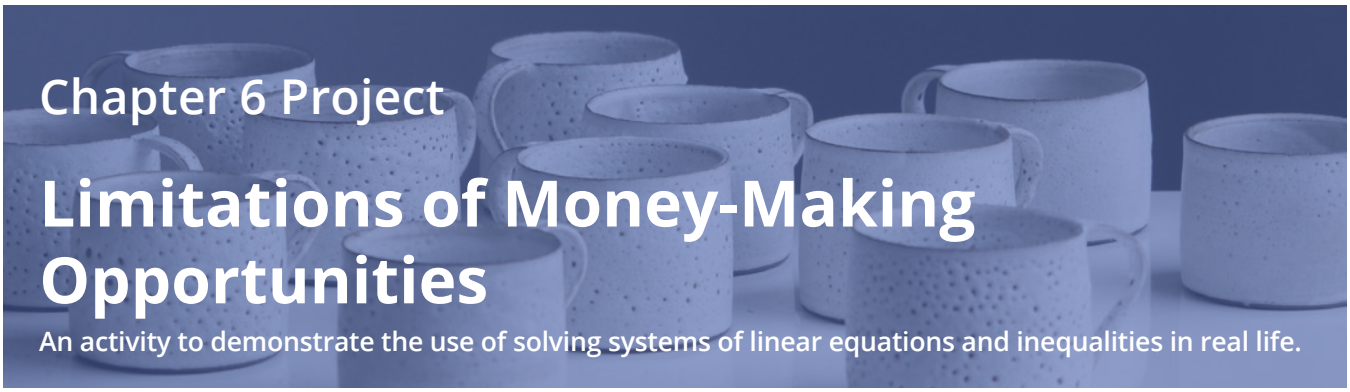

Chapter 6 Project

Limitations of Money-Making Opportunities

An activity to demonstrate the use of solving systems of linear equations and inequalities in real life.

In this project, you will investigate how limitations (known as *constraints*) on the production of goods can lead to limitations on how much profit a company can make.

Suppose the Soaring Eagle Book Store wishes to produce two types of coffee mugs: Type A and Type B. Each Type A mug will result in a profit of $3, and each Type B mug will result in a profit of $2.75. Manufacturing one Type A mug requires 6 minutes on Machine I and 3 minutes on Machine II. Manufacturing one Type B mug requires 4 minutes on Machine I and 6 minutes on Machine II. According to the schedule, Machine I has 12 hours available and Machine II has 10 hours available to make the mugs. How many of each type of mug should the bookstore make to maximize its profit?

1. Using the variables x and y, identify the two unknown quantities.

2. Consider the information provided about the time available on each machine. This information allows us to write two linear inequalities in the standard form $Ax + By \leq C$. These are called constraints.

- **a.** Explain what each constraint represents.
- **b.** Explain why the constraints should be of the less-than-or-equal-to type.
- **c.** Write the two constraints. (**Note:** Be mindful of how the times are expressed, minutes versus hours. You'll need to convert to make units consistent throughout.)

3. Regardless of maximizing profit, there are two more "common sense" constraints. Consider whether there is a minimum or maximum number of mugs. Using this information, write two more constraints and explain why they are constraints.

4. Graph the system of linear inequalities resulting from the four constraints. (**Hint:** You may wish to first graph the two inequalities from Problem 3, shading lightly. Once you've identified the intersection, erase extra shading. Then add the other two constraints found in Problem 2 to the graph and, once you've identified the new (smaller) intersection, erase extra shading again.)

5. The final result of graphing the four constraints should be a four-cornered region. Use systems of equations to solve for the (x, y) coordinates of the four corners.

6. Recall the profit that the bookstore can make from each type of mug. Write an equation of the form $P = Ax + By$, with A and B filled in but not P.

7. Consider the following six values of P: -400, -200, 0, 200, 400, and 600. Using your answer to Problem 6, lightly graph six lines for these six different values of P over the shaded four-cornered region from Problem 5. (**Hint:** These six lines will be parallel.)

8. Would all of the lines created in Problem 7 indicate a possible value of P? Explain why or why not.

9. Is there a maximum value of P or can it keep growing? Why or why not?

10. Let's determine the maximum profit that Soaring Eagle Book Store can make from selling mugs.

- **a.** If there is a maximum value of P, at what point does it seem to occur?
- **b.** Is there anything special about the point found in part a.?
- **c.** Substitute this point into your $P = Ax + By$ equation from Problem 6 to calculate the maximum profit. Express your final answer in words; that is, state how many mugs of each type should be made to produce the maximum profit and what that maximum profit is.