Chapter 11 Project

Determining Product Pricing to Maximize Revenue

An activity to demonstrate the use of quadratic equations in real life.

A company determines that if p is the price charged for an electric bicycle they manufacture, then the number of bikes that will sell n is a function of p: $n(p) = 3750 - 3p$. This is because for each \$1 increase in the price, three fewer bikes are sold. The revenue R earned is also a function of p because it is the number of bikes sold times the price per bike: $R(p) = p(3750 - 3p) = -3p^2 + 3750p$. Using methods learned in this chapter, we will investigate just how much revenue the bike manufacturer can earn. Is it boundless or is there a maximum?

For simplicity, in order to compare variable names used within the chapter, let's replace p with x and R with y, so $y = x(3750 - 3x)$. Notice that the graph of this equation is a parabola.

1. Solve the equation $x(3750 - 3x) = 0$ for x. Notice that by setting $y = 0$, you are solving for the x-coordinates of the two x-intercepts. State the coordinates of the two x-intercepts.

2. Use the equation $y = x(3750 - 3x)$ for the following.
   - **a.** Choose an x-value smaller than the smallest x-value of the x-intercepts found in Problem 1 and calculate y.
   - **b.** Choose an x-value larger than the largest x-value of the x-intercepts found in Problem 1 and calculate y.
   - **c.** What do you notice about y-values found in parts a. and b? Recall that y is revenue and x is price. Can you conclude an initial interval for price that the company should stay within?

3. Calculate the mean of the two x-values found in Problem 1.

4. Pick two new values of x between the smallest x-value from Problem 1 and the answer to Problem 3. Next, pick two values of x strictly between the answer to Problem 3 and the largest x-value from Problem 1.

5. Arrange these four values from least to greatest and include your answer from Problem 3, for a total of five unique values.

6. **a.** Evaluate $y = x(3750 - 3x)$ for these five values of x from Problem 5. This will give you five points on the parabola.
   - **b.** For the smallest of the x-values from part a, explain in words what the (x, y) coordinates represent. Include the values.

7. Carefully choose a horizontal and vertical scale and plot the five points found in Problem 6 part a. Also plot the x-intercepts found in Problem 1. Use these points to sketch the parabola.

8. Based on your values and the graph, state the (x, y) coordinates of the vertex (or your best approximation).

9. Is the y-coordinate of the vertex a minimum or maximum value of y? Why do you think this is, based both on the context and on the function?

10. Use the vertex formula and the equation $y = x(3750 - 3x) = -3x^2 + 3750x$ to find the vertex of the parabola.

11. Does the vertex found in Problem 10 support your approximation of the vertex found in Problem 8?

12. Now return to the question about revenue from selling electric bicycles.
    - **a.** What is the lower bound on how much revenue the company can earn?
    - **b.** What is the upper bound on how much revenue the company can earn? What price is charged for that to be the revenue earned?
    - **c.** How many bikes need to be sold to reach the maximum revenue found in part b? (**Hint:** Recall the number of bikes sold is given by $n(p) = 3750 - 3p$ where p is the price per bike.)