8.3 EXERCISES

PRACTICE

For each of the Exercises 1–24, find all local maxima, local minima, and saddle points.

1. $f(x,y)=x^2+y^2-6x+2y-4$

2. $f(x,y)=x^2+2y^2+8x-4y+2$

3. $f(x,y)=x^2-y^2-10x+2y+9$

4. $f(x,y)=12x+8y-x^2-y^2-7$

5. $f(x,y)=5x+8y-x^2-y^2+11$

6. $f(x,y)=y^2-x^2+6x-y-10$

7. $f(x,y)=x^2+xy-y^2-5y-8$

8. $f(x,y)=x^2-2xy+4y^2-6y+3$

9. $f(x,y)=2x^2-3xy+3y^2+5x-13$

10. $f(x,y)=10x-2x^2+2xy-3y^2+5$

11. $f(x,y)=x^2-xy+y^2-2x-2y+1$

12. $f(x,y)=3x^2-2xy+y^2-16x+4y+14$

13. $f(x,y)=-x^2+xy-y^2+4x-5y+6$

14. $f(x,y)=x^2+3y^2+5xy+4x-3y+15$

15. $f(x,y)=x^3+y^2-3x-6y+11$

16. $f(x,y)=x^3-3x^2-2y^2-9x+8y+7$

17. $f(x,y)=x^2-y^3+9y^2-4x-15y-14$

18. $f(x,y)=2x^2+y^3-3y^2-12x-24y+21$

19. $f(x,y)=x^3-3x^2y+12y$

20. $f(x,y)=9x-xy^2+2y^3$

21. $f(x,y)=2x^2-x^2y+y^2$

22. $f(x,y)=x^2-2xy^2+4y^2$

23. $f(x,y)=xy+\frac{2}{x}+\frac{4}{y}$

24. $f(x,y)=xy+\frac{9}{x}+\frac{3}{y}$

APPLICATIONS

25. Profit: An automobile agency sells two models of a car. The annual profit is estimated by $P(x,y)=-0.1x^2-0.2y^2+6x+10y-160$ in thousands of dollars. Find the number of each model that should be sold to maximize profit.

26. Sales: The owner of a small business advertises in the newspaper and on radio. He has found that the number of units that he sells is approximated by $N(x,y) = -0.5x^2 - y^2 + 8x + 12y + 240$, where x (in thousands of dollars) is the amount spent on newspaper advertising and y (in thousands of dollars) is the amount spent on radio advertising. How much should he spend on each to maximize the number of units sold?

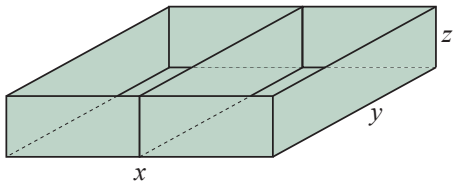
27. Profit: A firm produces two kinds of magazine racks, one selling for \$50 and the other for \$45. The total cost of producing x of the \$50 racks and y of the \$45 racks is given by $C(x,y) = 0.15x^2 + 0.1y^2 - 10x - 3y + 4760$ dollars. Find the number of each kind that should be produced and sold to maximize the profit.

28. Profit: A company makes two types of work gloves, leather and cloth. The leather gloves sell for \$5.80 and the cloth gloves for \$1.60. The total cost function is $C(x,y) = 0.25x^2 + 0.03y^2 + 1.3x + 0.4y + 14$ in thousands of dollars, where x (in thousand pairs) is the number of leather gloves and y (in thousand pairs) is the number of cloth gloves. How many gloves of each type should be produced and sold to maximize profits?

29. Revenue: A department store sells two types of T-shirts, adult and youth. The store manager has determined that he can sell $23 - 6x + 8y$ adult T-shirts and $26 + 5x - 9y$ youth T-shirts if the price is x dollars for the adult and y dollars for the youth. Find the price of each that will yield maximum revenue.

30. Revenue: A grocery store sells two brands of a product, a name brand and a store brand. The manager estimates that if she sells the name brand for x dollars per unit and the store brand for y dollars per unit, she will be able to sell $62 - 20x + 18y$ units of the name brand and $53 + 16x - 22y$ units of the store brand. Find the price of each that will yield maximum revenue.

31. Construction: A rectangular box is to be constructed without a top and with one partition. The volume of the box must be 162 in.^{3} Find the dimensions that will minimize the material required to construct the box.



32. Packaging: The Postal Service has a limit of 108 inches on the combined length and girth of a rectangular package to be sent by parcel post. Length is the measurement for the longest side and girth is the distance around the package perpendicular to the length. Find the dimensions of the package of maximum volume that can be sent.